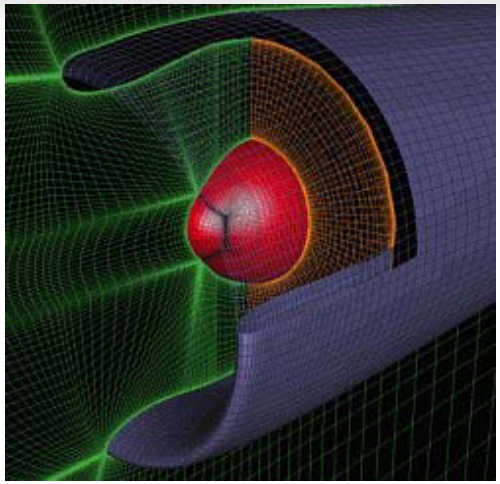
An Introduction to Mesh Generation Algorithms
Part 2

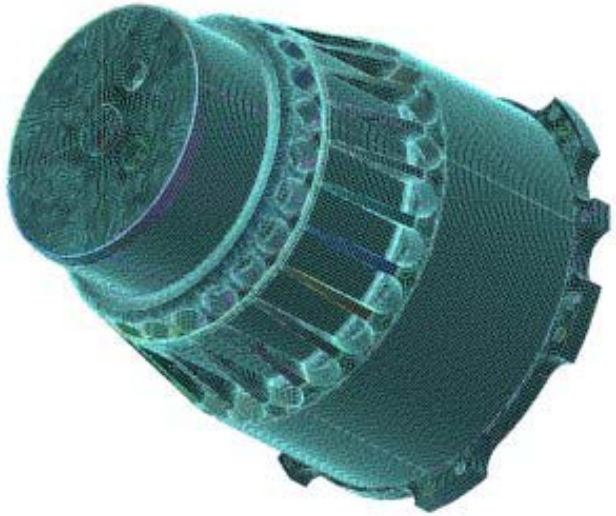
Fathi El-Yafi
Senior Product Engineer
Product Department of EXA Corporation

Overview

Adaptive Meshing:

- Remeshing
- Decimation
- Optimization

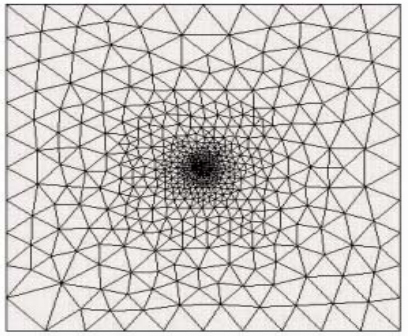




Adaptive Meshing

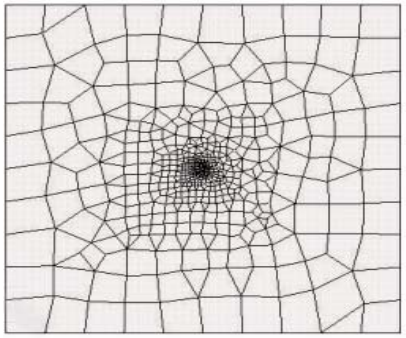
Objective:

2D Adaptive Mesh according to the specifications sizes imposed by the user



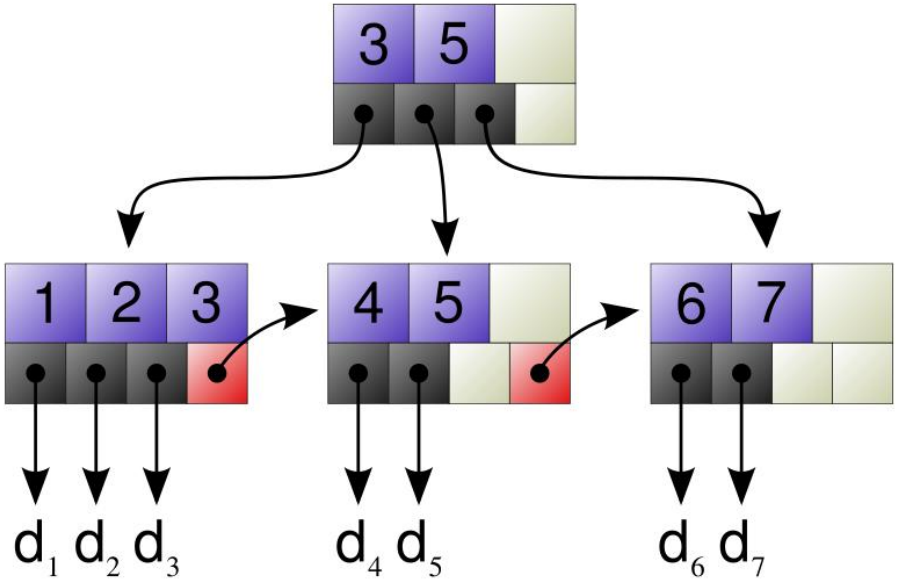
Method:

- Calculating the density of the mesh from a tree Quaternary
- Nodal generation within each cell of the tree
- Meshing by advancing front method according to a criterion of Delaunay



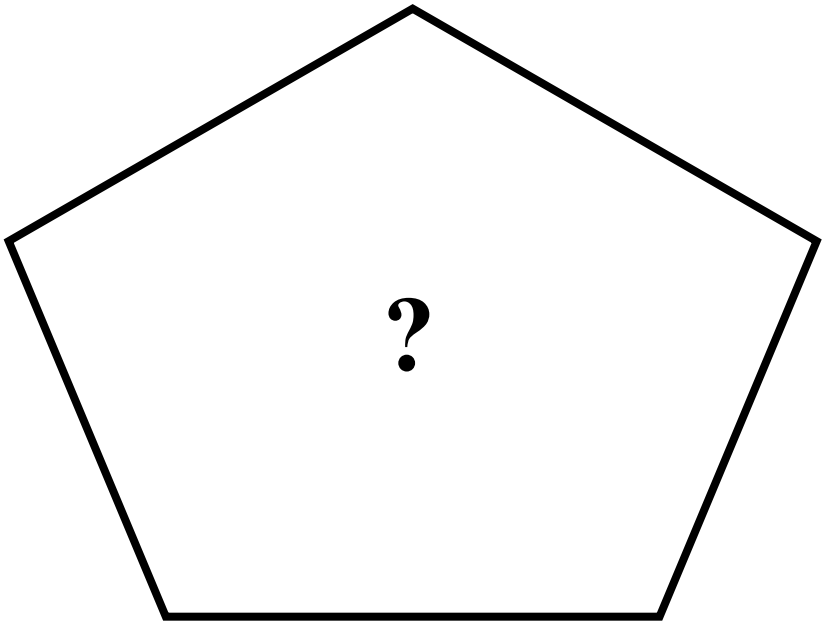
Finite Element Context

Quaternary Tree



In computer science, a **B+ tree** (also known as a **Quaternary Tree**) is a type of tree, which represents sorted data in a way that allows for efficient insertion, retrieval and removal of records, each of which is identified by a *key*. It is a dynamic, multilevel index, with maximum and minimum bounds on the number of keys in each index segment (usually called a block' or node'). In a B+ tree, in contrast to a B+ tree, all records are stored at the lowest level of the tree; only keys are stored in interior blocks.

Remeshing a Polygon

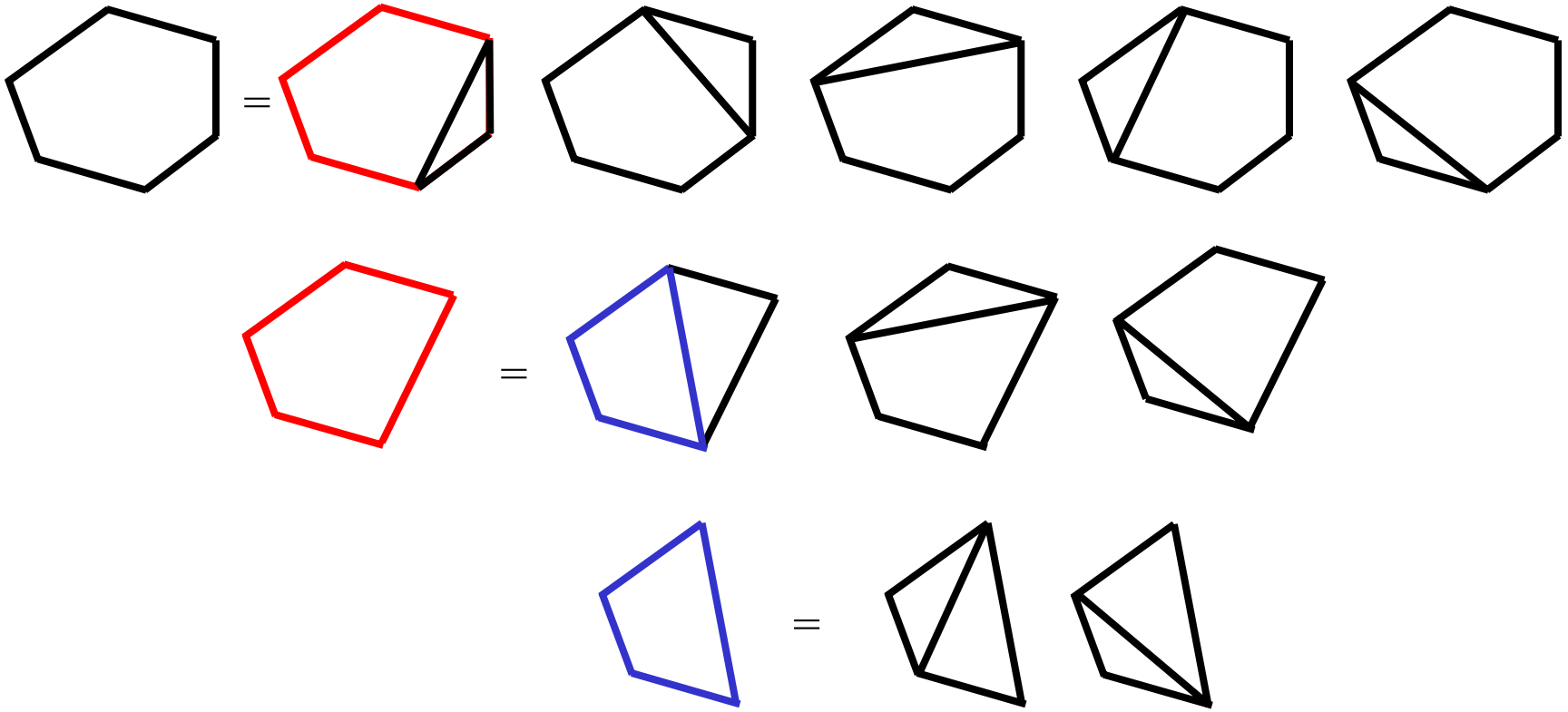


n points

Possible Remeshing?

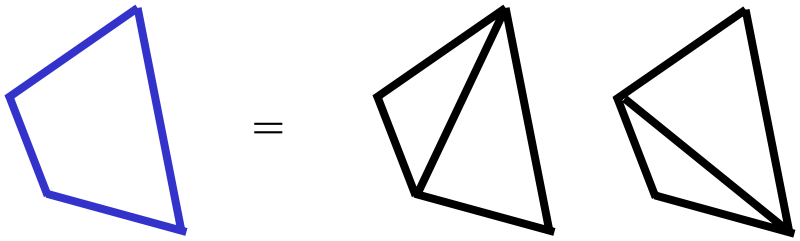
Finite Element Context

Principle



Principle

n nodes lead to n-2 triangles



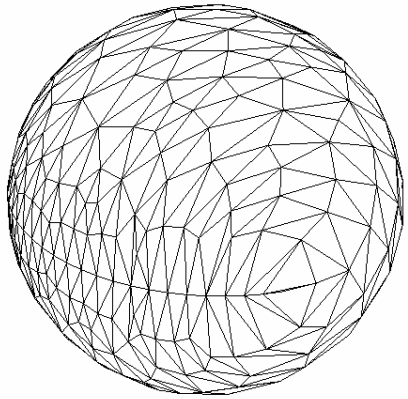
It shows that we can create Tn different triangles

$$T_n = \frac{n(n-1)(n-2)}{6}$$

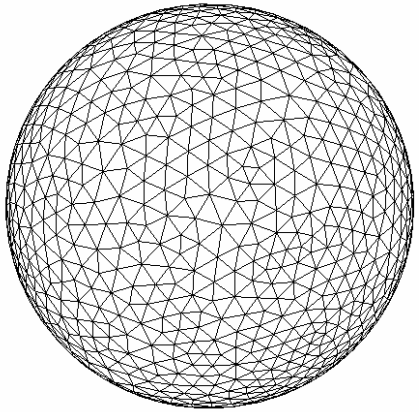
3	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	**4**	**10**	**20**	**35**	**56**	**84**	**120**

N=10, 1430 possible triangulations!

State of the Art



- **Methods that use the surface parameterization:**
  - Appropriate Plane + transport functions
  - Appropriate Plane + Metric: INRIA



- **Discrete Models:**

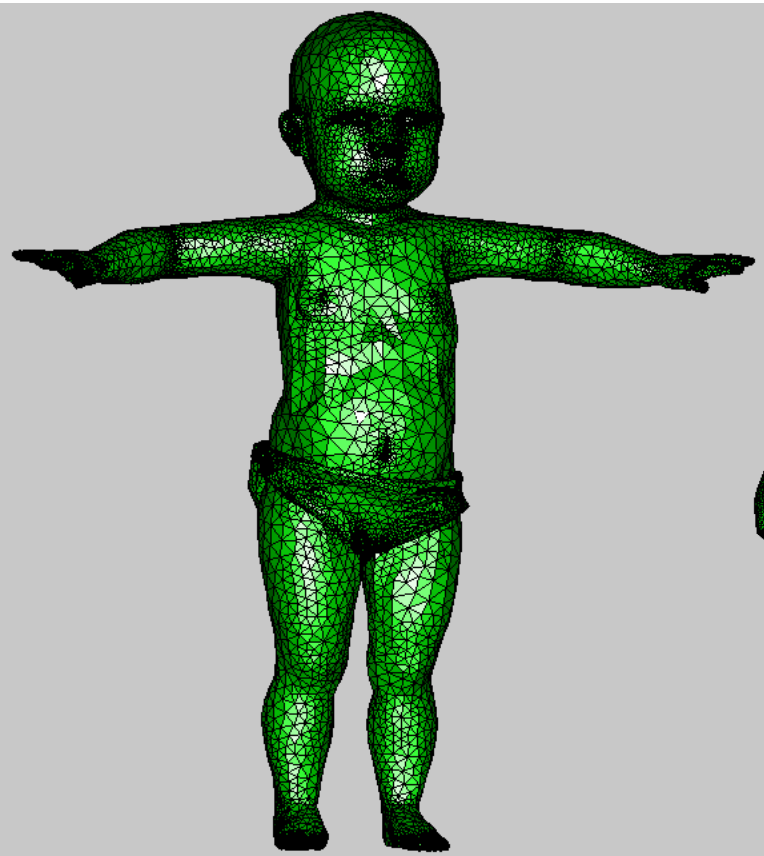
  Very Refined Initial Mesh + Simplifications

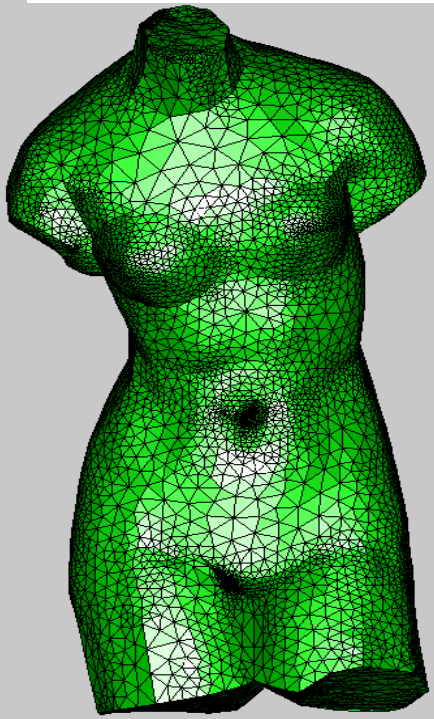
  Reconstruction:

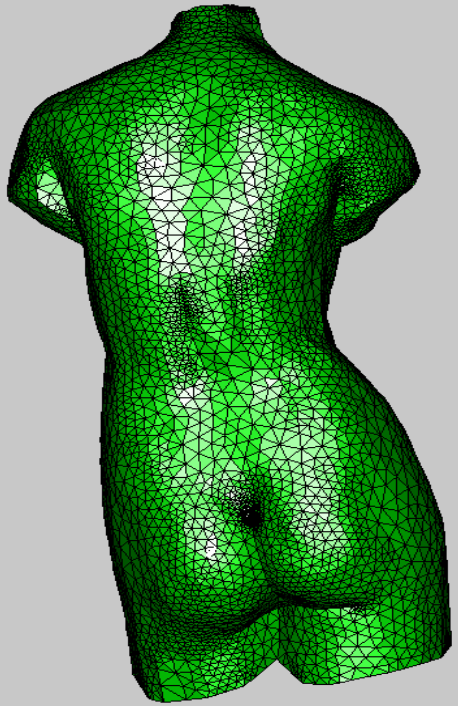
  - Direct Method: Frontal Triangulation (R. Löhner)
  - Remeshing: HDI (A. Rassineux)

Adaptive Mesh

Typical Examples



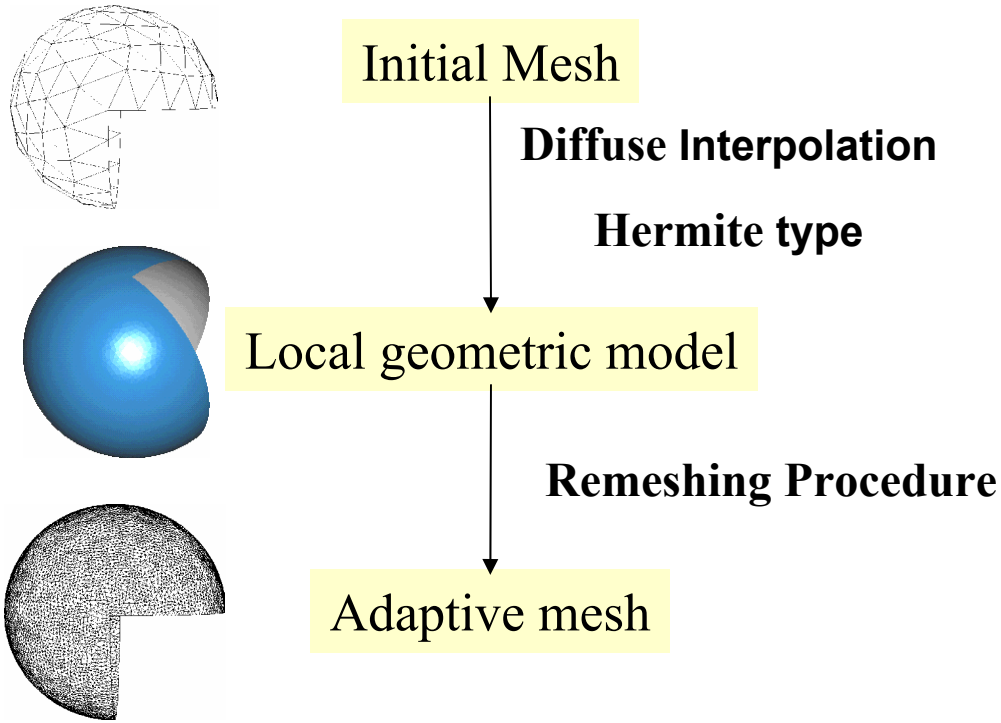




Adaptive Meshing using Diffuse Interpolation

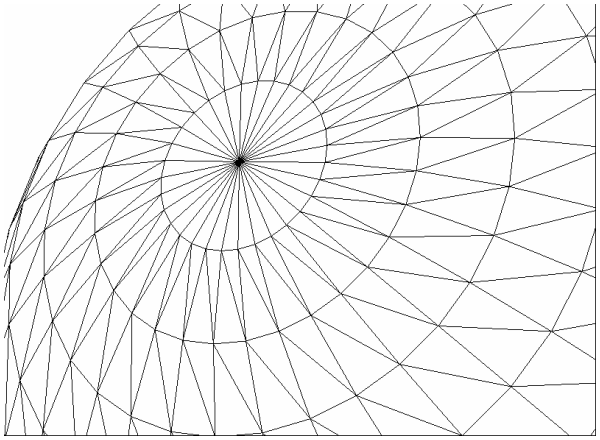
Objective:

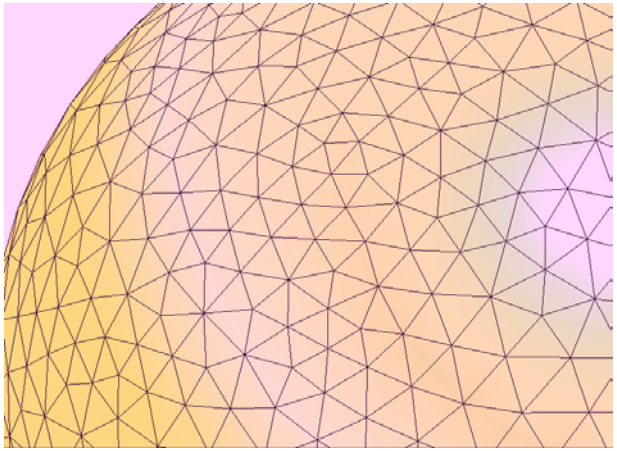
Building an adaptive mesh of a 3D surface according to size map, using the initial mesh as only data

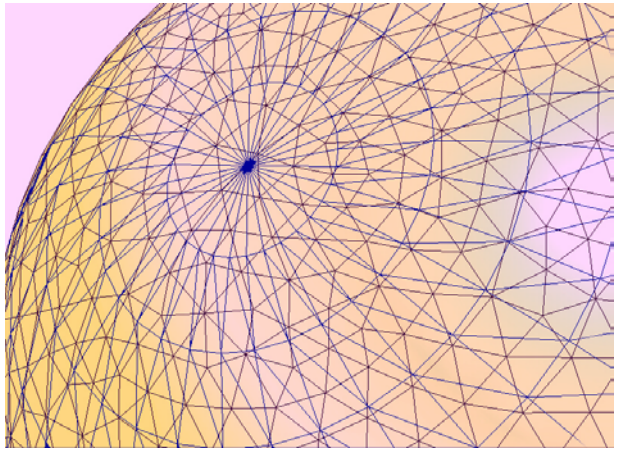


Adaptive Mesh

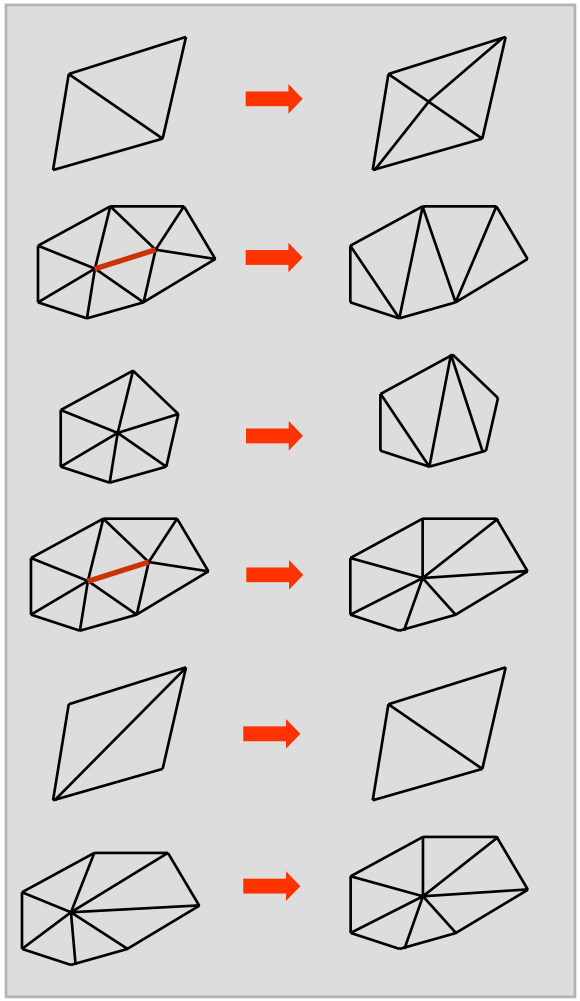
Remeshing







Some Remeshing Tools



Edge splitting

Edge collapsing

Node removing

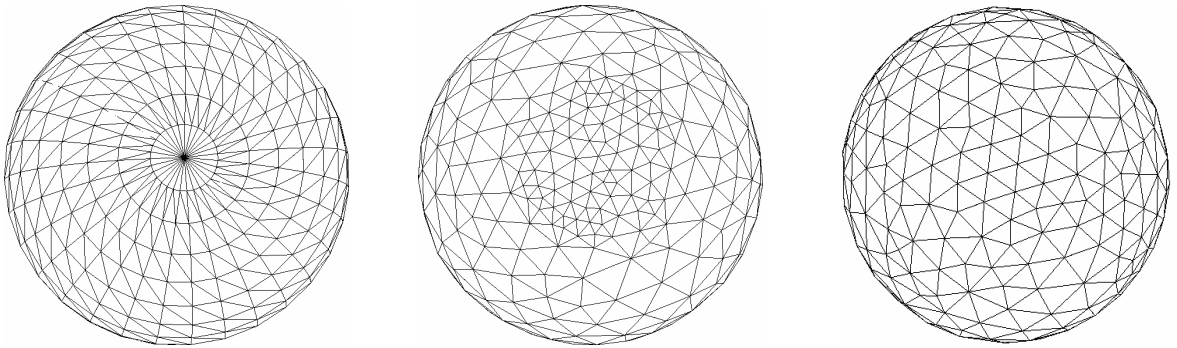
Edge collapsing and nodal insertion

Swapping

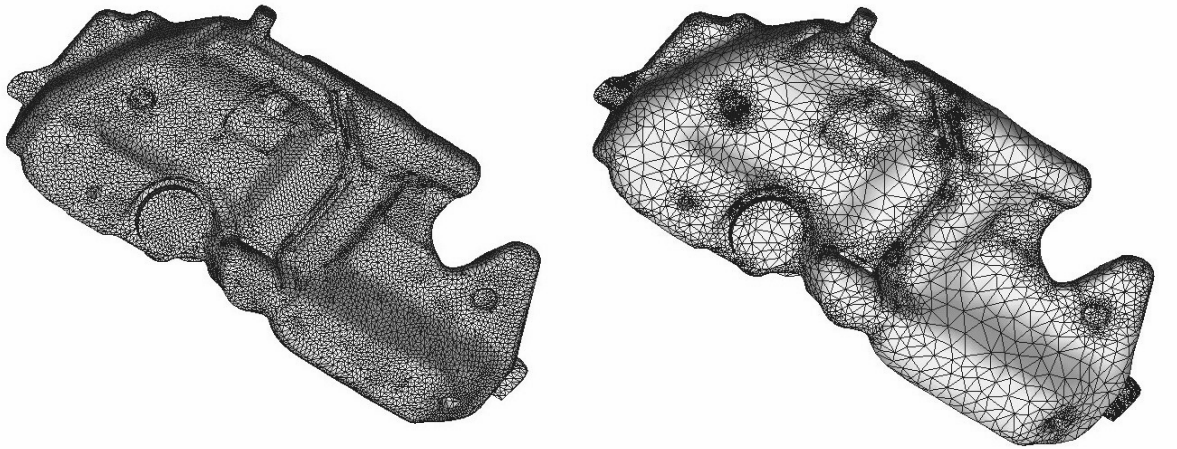
Nodal Shifting

Adaptive Mesh

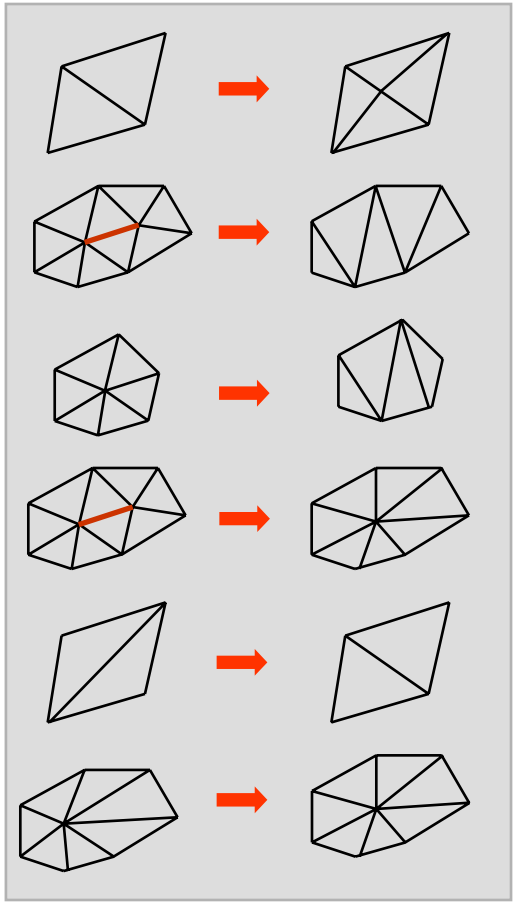
Remeshing



Remeshing according to a size map

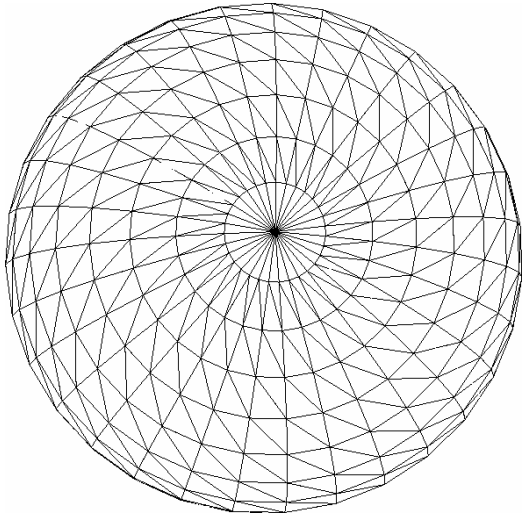


Simplification of a mesh (70%)

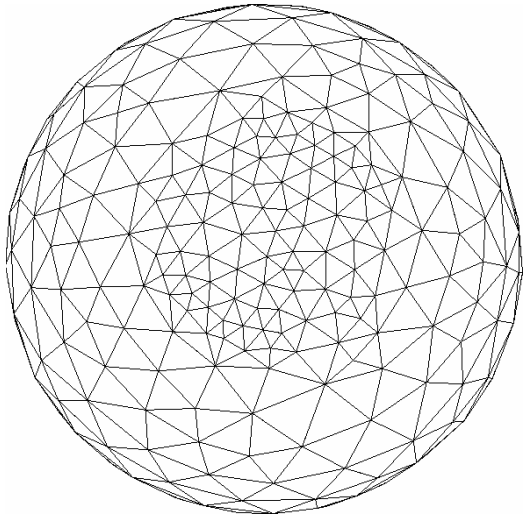


Adaptive Mesh

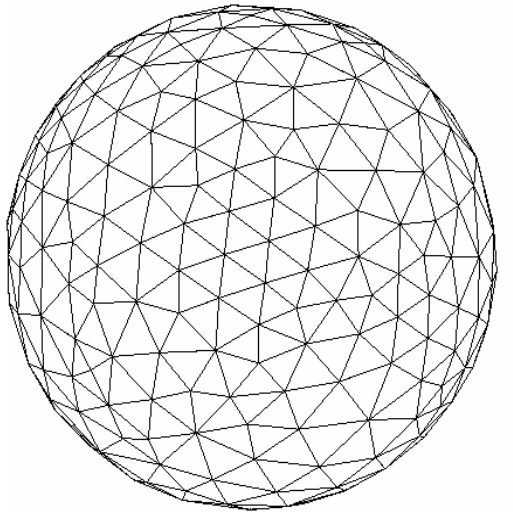
Remeshing



Initial Mesh



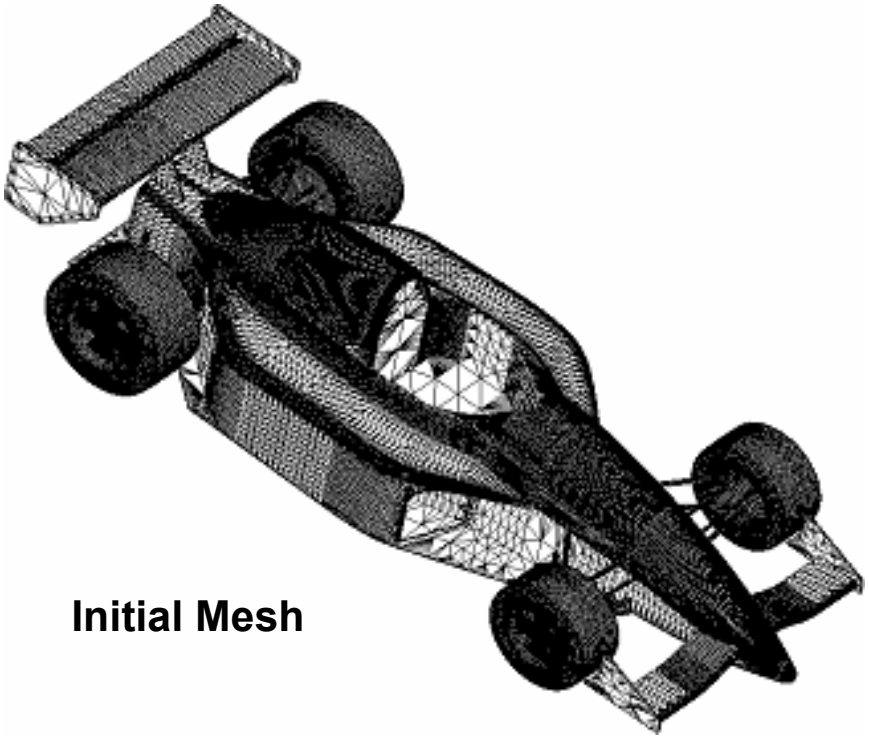
Intermediate Mesh



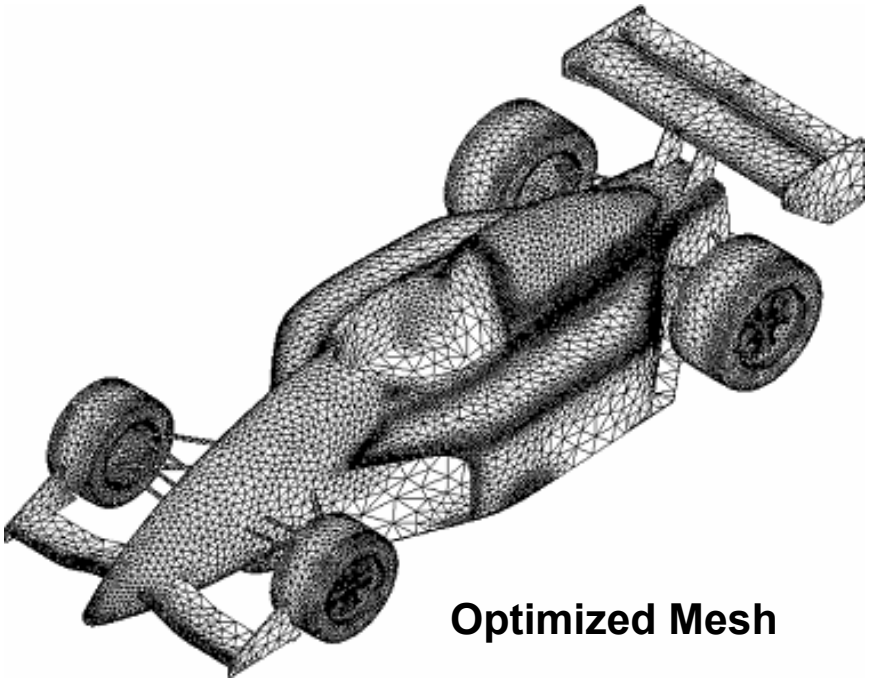
Optimized Mesh

Adaptive Mesh

Surface Remeshing



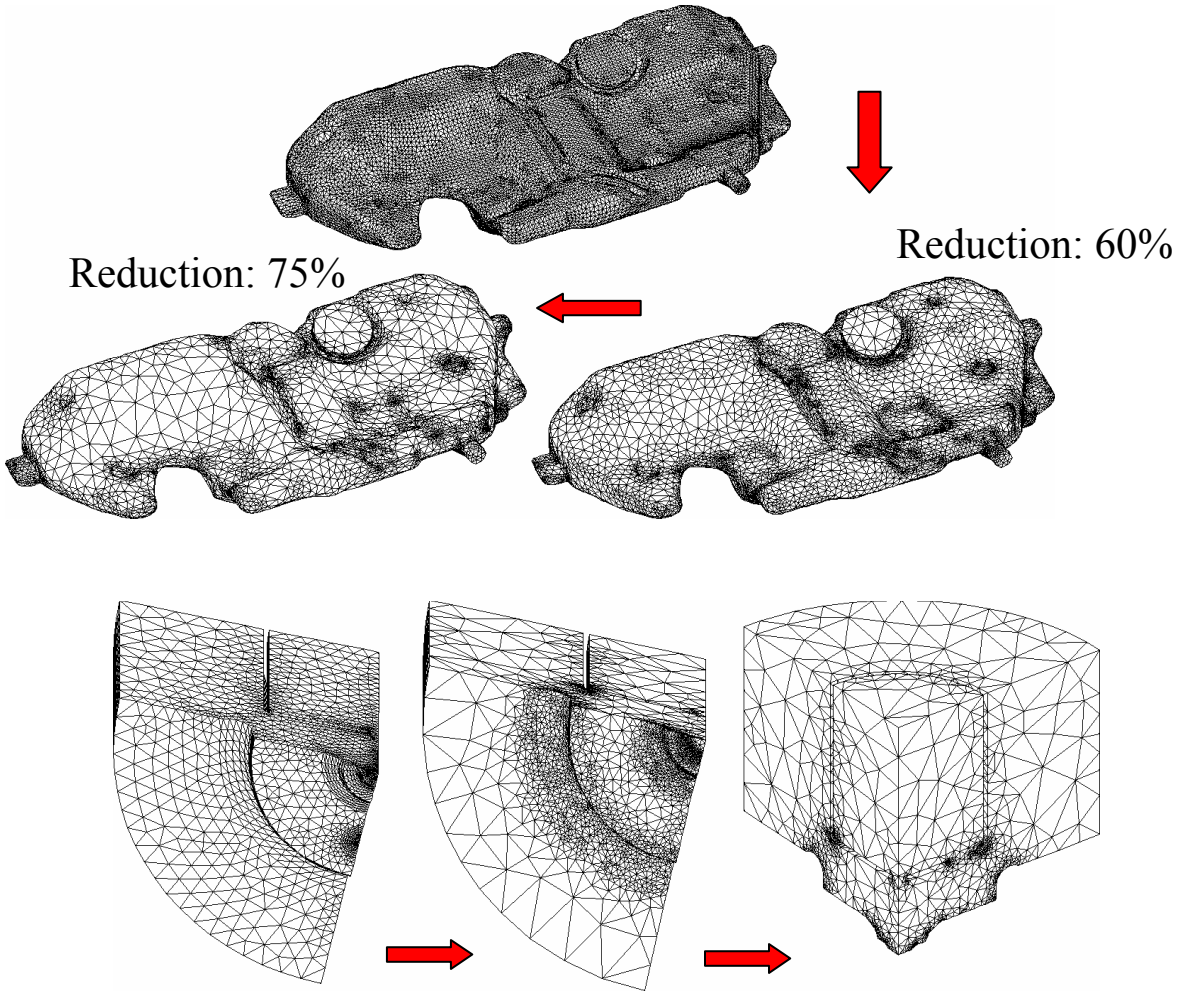
Initial Mesh



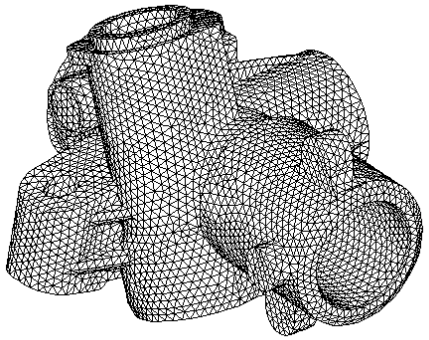
Optimized Mesh

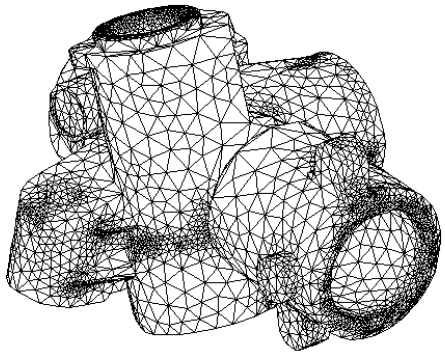
Adaptive Mesh

Surface Remeshing



Initial model

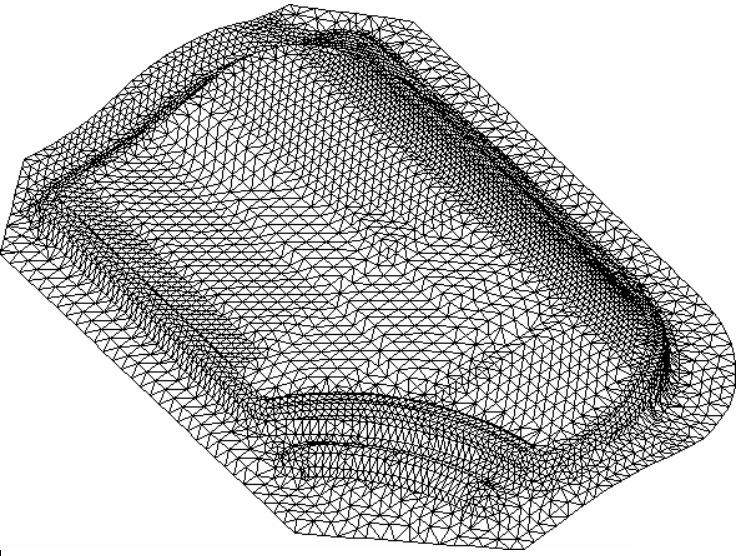


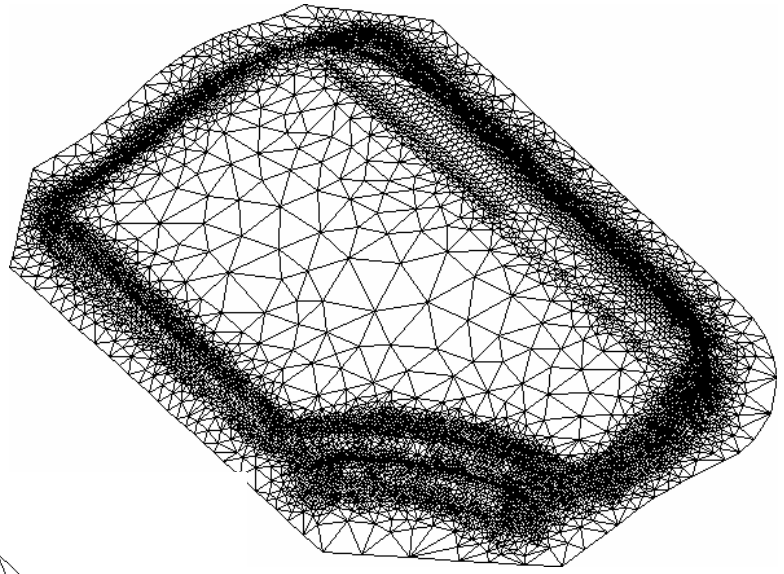


Adapted to curvature

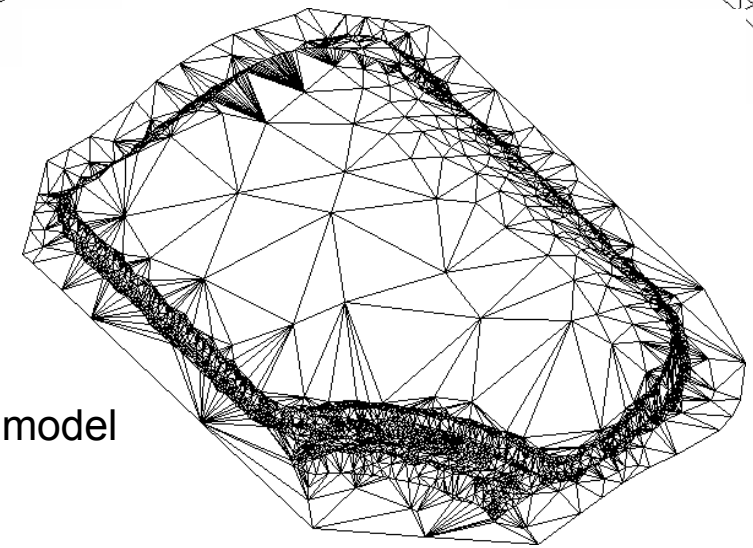
Adaptive Mesh

Surface Meshing





Adapted to curvature

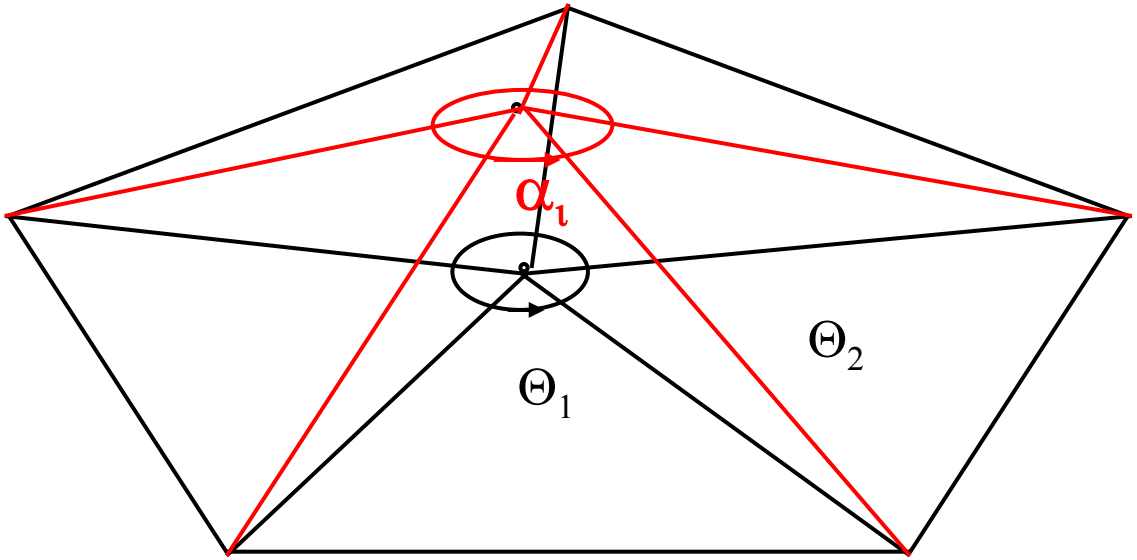


Simplified model

Adaptive Mesh

Simplified Curvature

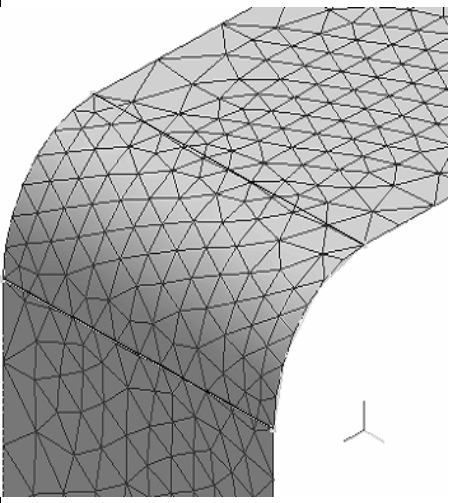
In a plane $\sum \theta_i = 360°$

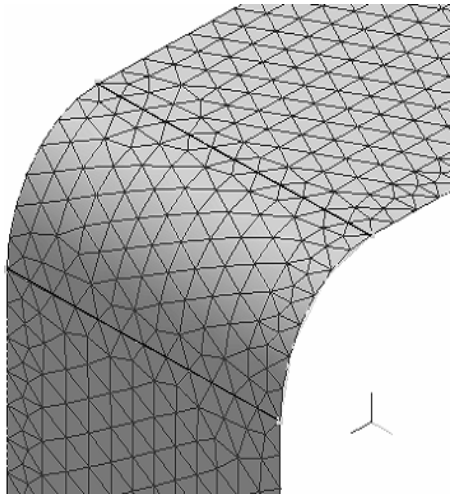


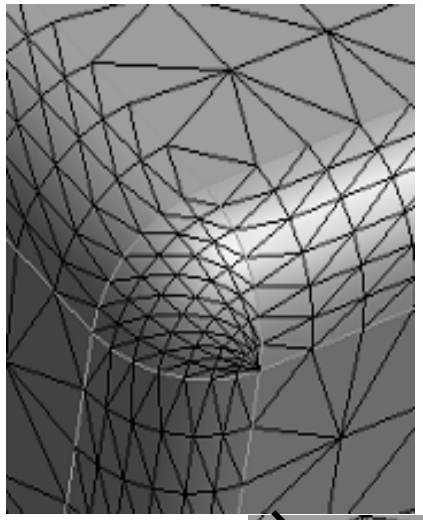
Of all the triangles $\sum \alpha_i \geq 360°$

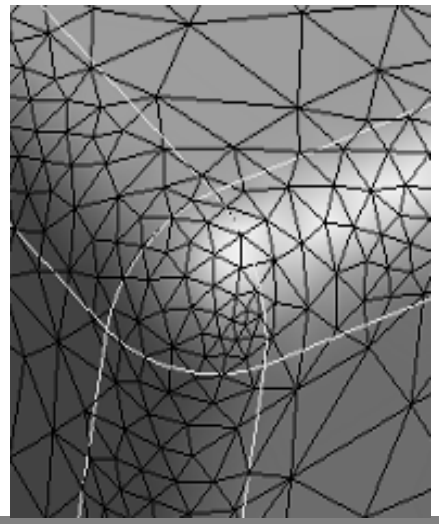
$\sum (\alpha_i - \theta_i) = \Delta\theta =$ default angle = f (Gaussian Curvature)

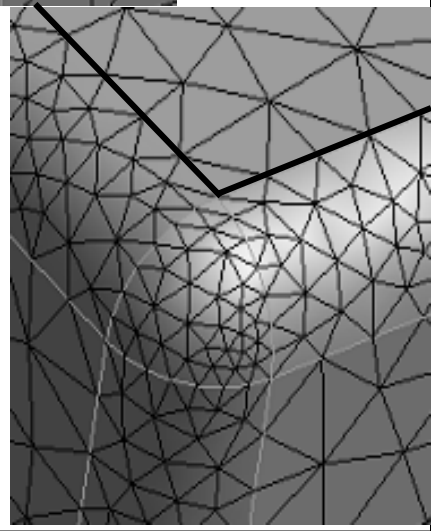
Feature Lines Detection



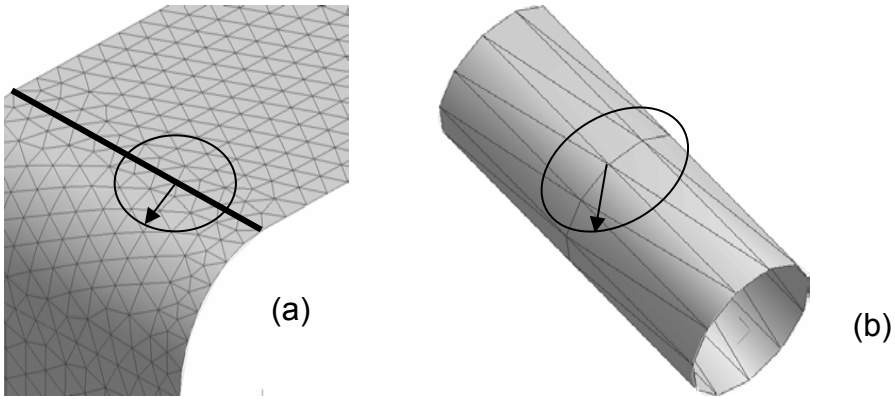








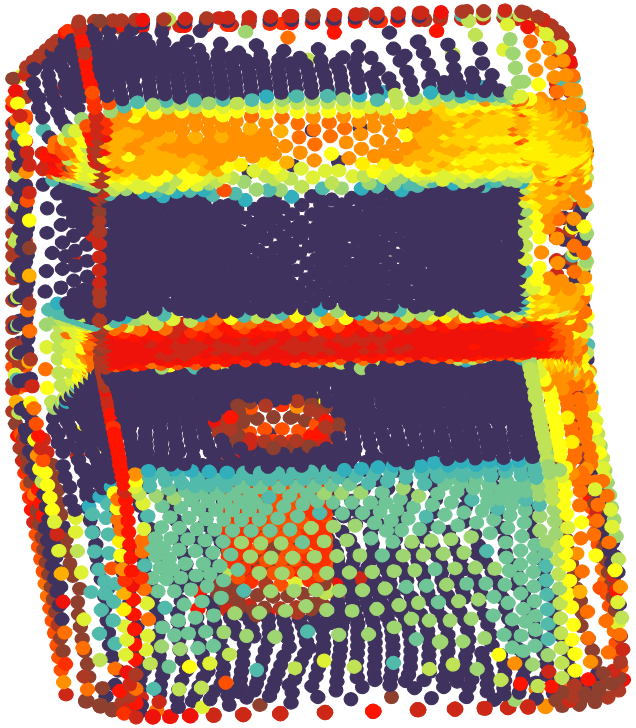
Feature Lines Detection

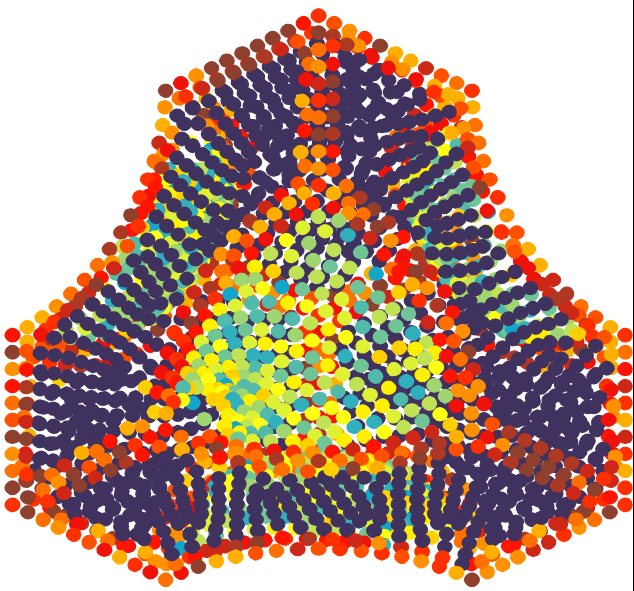


- Curvature using Diffuse Interpolation Model
- Detecting neighborhood
- Identification of geometrical parameters (radius, axis)

Principal Curvatures

- **Segmentation into subsets**
- Surface Identification
- Detection of Feature lines
- Meshing

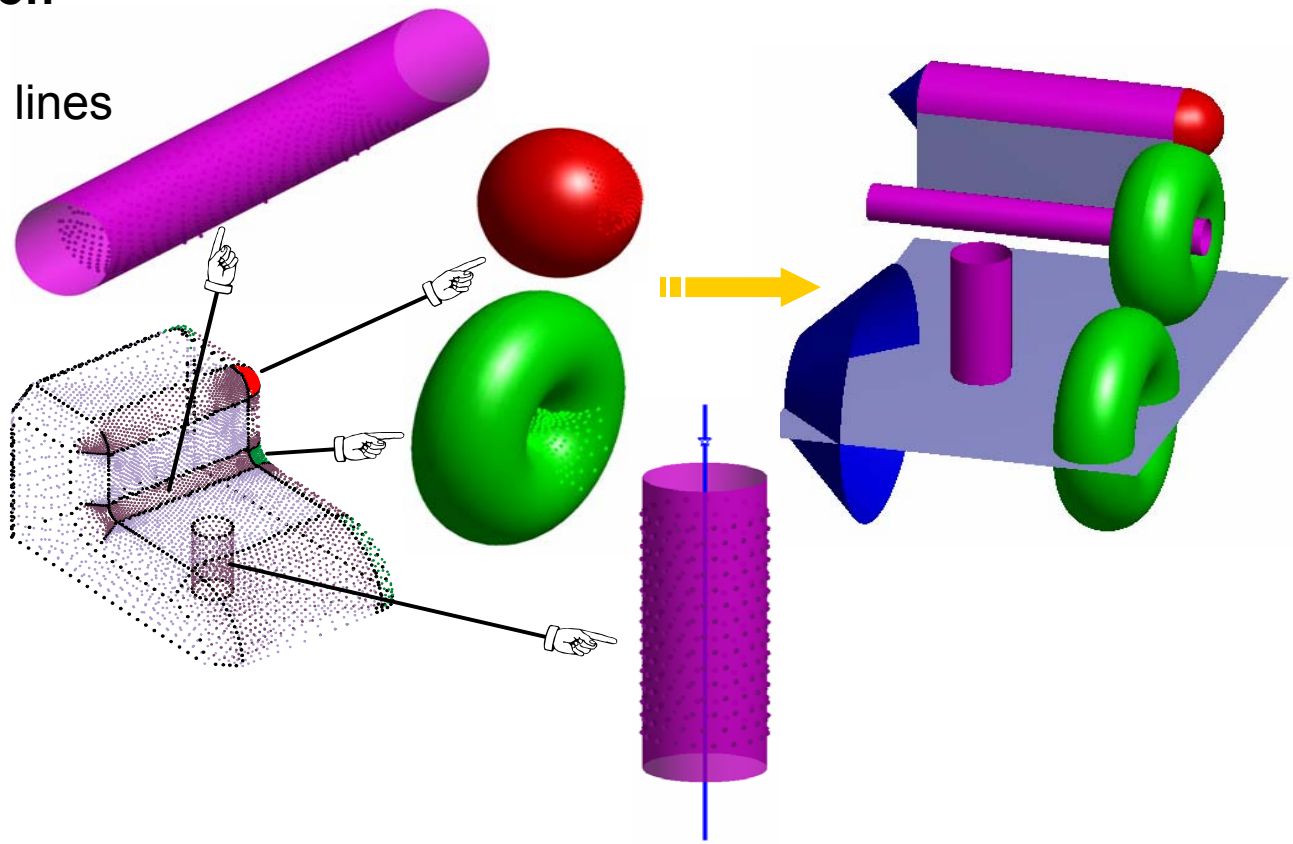




Adaptive Mesh

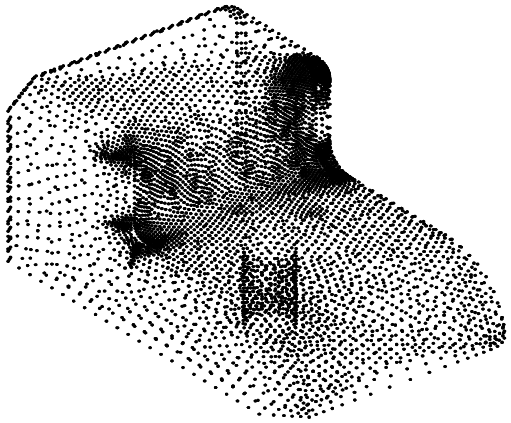
Example

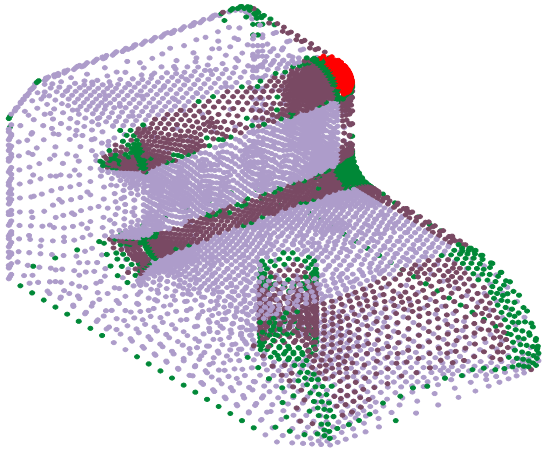
- Segmentation into subsets
- **Surface Identification**
- Detection of Feature lines
- Meshing

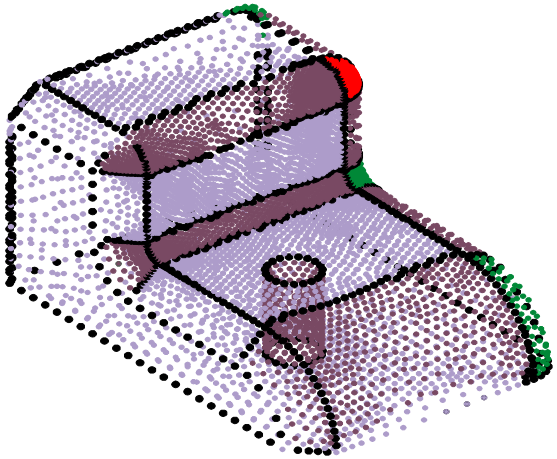


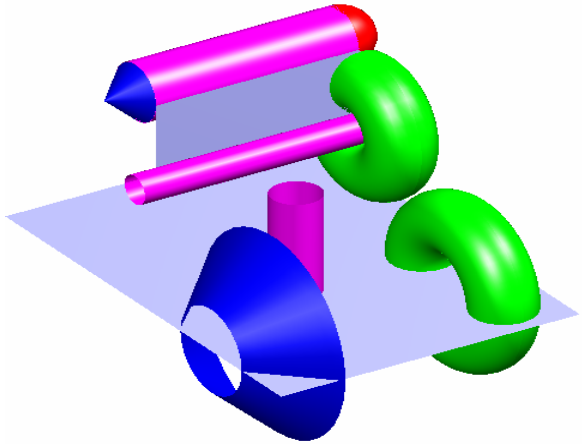
Adaptive Mesh

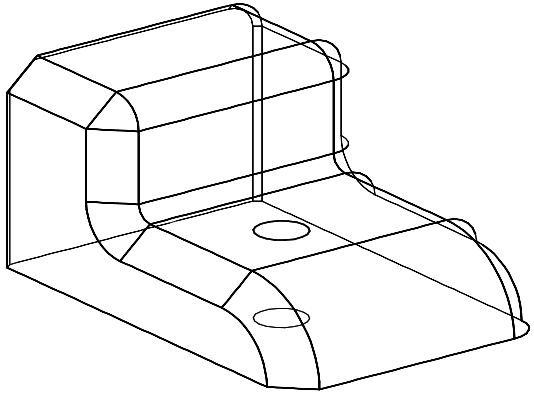
Main Steps

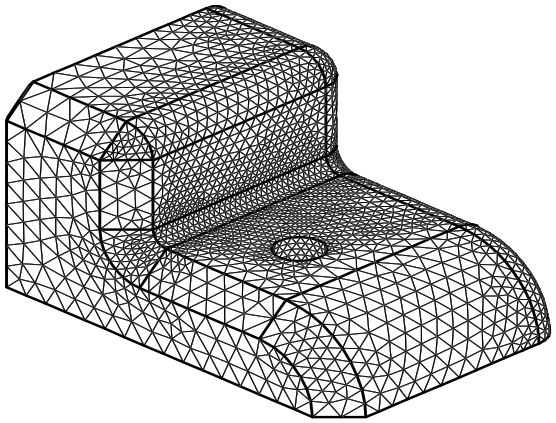




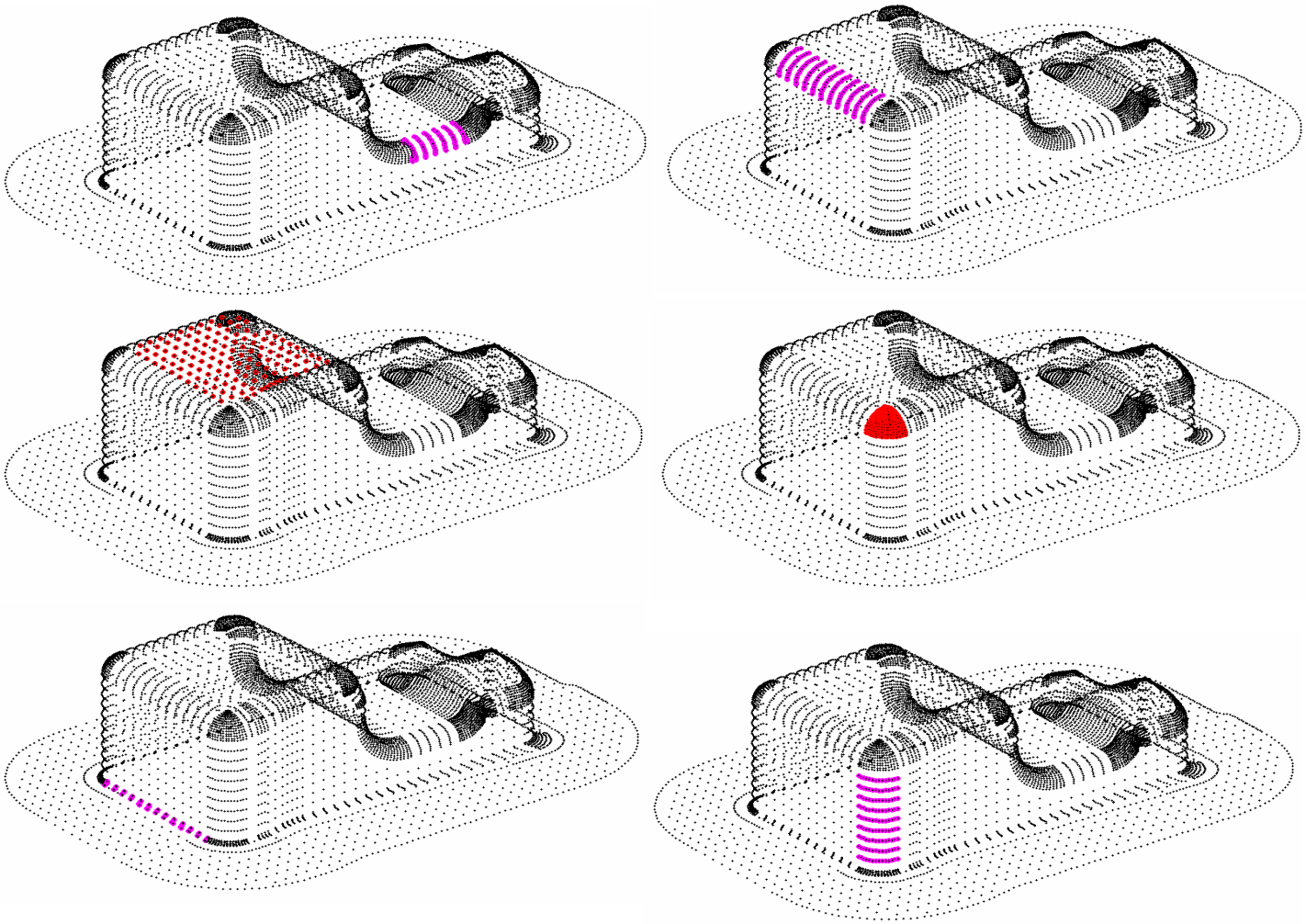






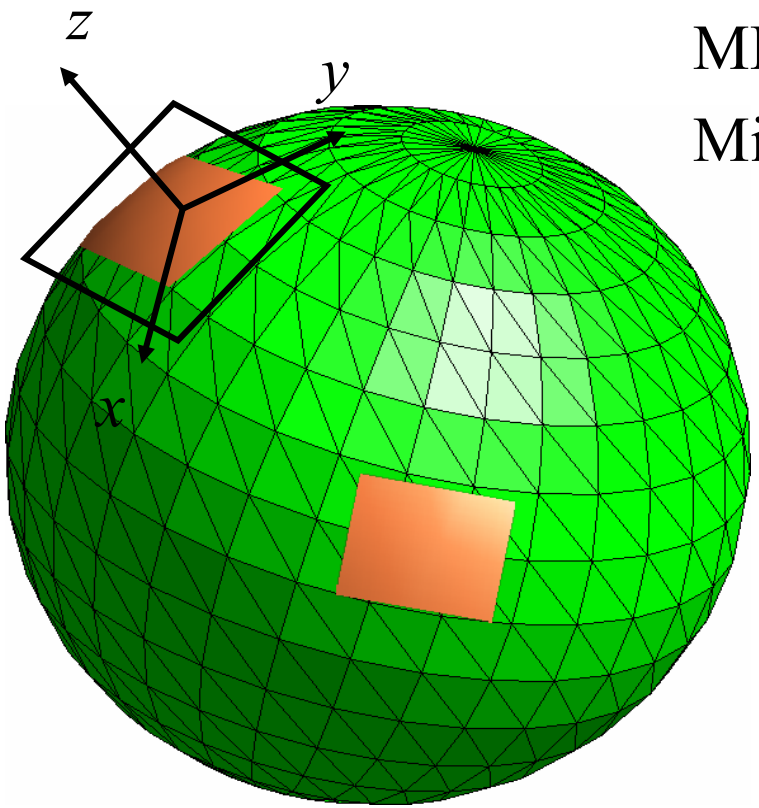


Adaptive Mesh



Diffuse Model

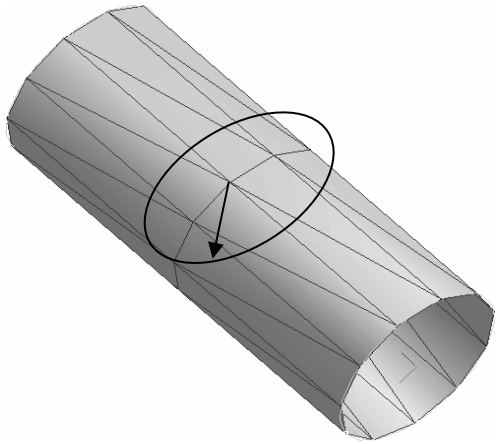
Local model $z = f(x,y) =< 1, x, y, x^2, xy, y^2 > \alpha = P^T \alpha$



MLS Technique : Diffuse approximation

Minimisation of an interpolation criterion

$$J_{1x}(\alpha) = \sum_{i=1}^{i=n} w(x_i, x) \times \left(p^T (x_i - x)\alpha - z_i \right)^2$$



Moving Least Squares

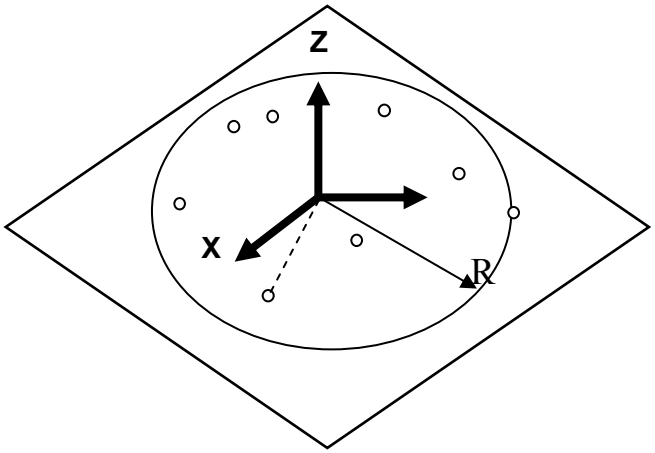
- Polynomial Base $< 1, x, y, x^2, xy, y^2 >= p^T(x,y) = p^T(x)$

n = Total number of points $\Rightarrow$ «standard» Least Squares

$$J_x(\alpha) = \sum_{i=1}^{i=n} \left(p^T(x_i - x)\alpha - f(x_i - x) \right)^2$$

- Select a neighbor *V(x)* around a point*(x,y)*

$$J_x(\alpha) = \sum_{i \in V(x)} \left(p^T(x_i - x)\alpha - f(x_i - x) \right)^2$$

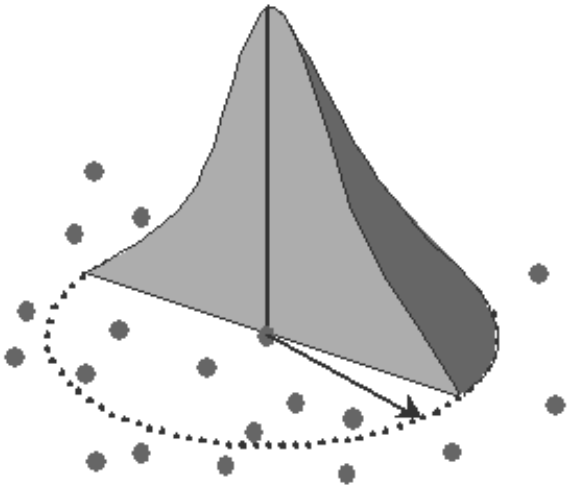


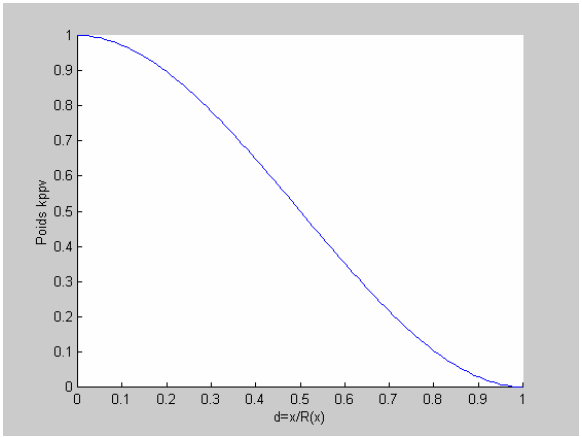
$\Rightarrow$ The approximation is local

Weighted Moving Least Squares

Weight Function

$$J_x(\alpha) = \sum_{i \in V(x)} w(x_i - x)\left(p^T (x_i - x)\alpha - f(x_i - x)\right)^2$$





$$w(x_i,x) = w_{ref}\left(\frac{\|xi-x\|}{R(x)}\right)$$

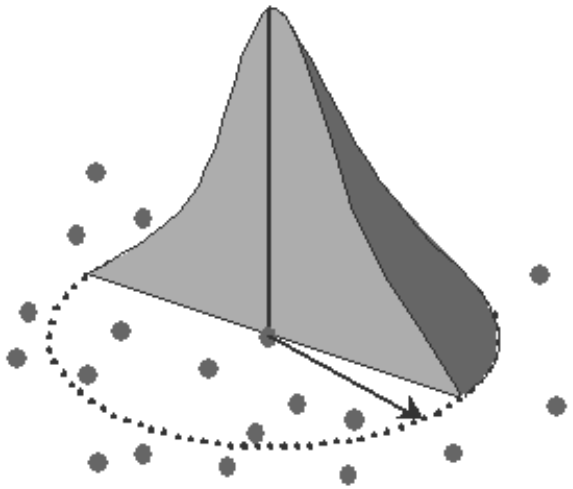
$$w(x_i,x) = (1-l_i)^2 \times (1+2l_i)$$

avec $l_i = d_i/R$

Moving Least Squares Interpolation

$$J_x(\alpha) = \sum_{i \in V(x)} w(x_i - x)\left(p^T (x_i - x)\alpha - f(x_i - x)\right)^2$$

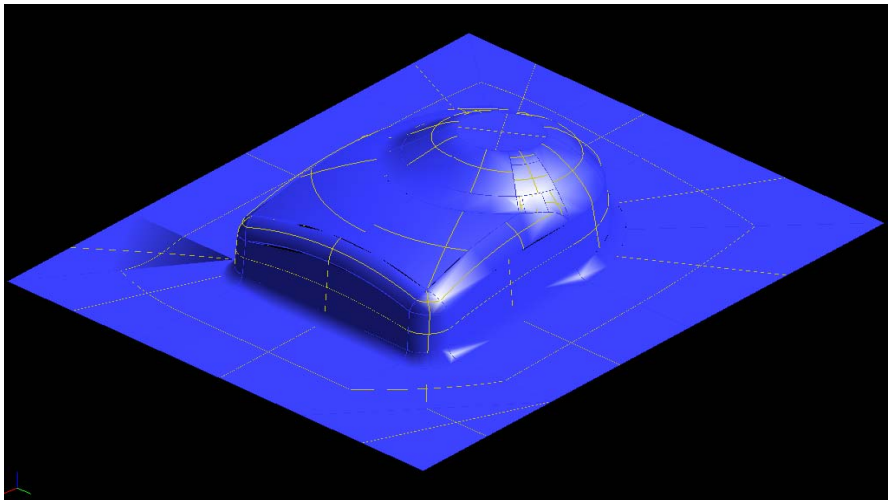
If $w(x,x_i) \to \infty \Rightarrow$ interpolation at point x_i

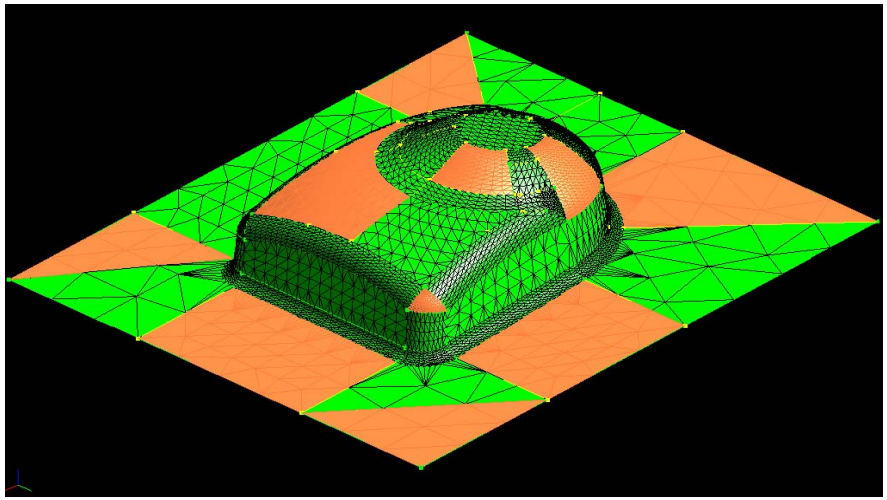


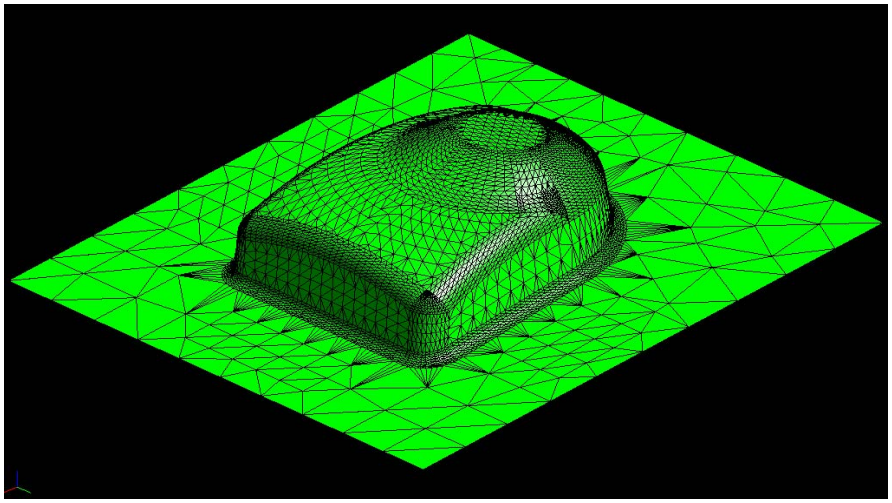
$$[w(x_i, x)] \to \left[\frac{w(x_i, x)}{1 - w(x_i, x)}\right]$$

Adaptive Mesh

CAD-STL Face Per Face

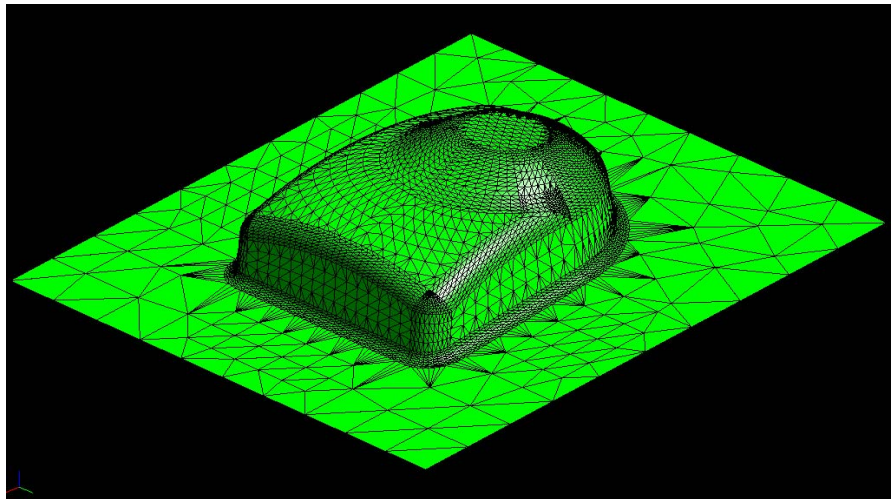




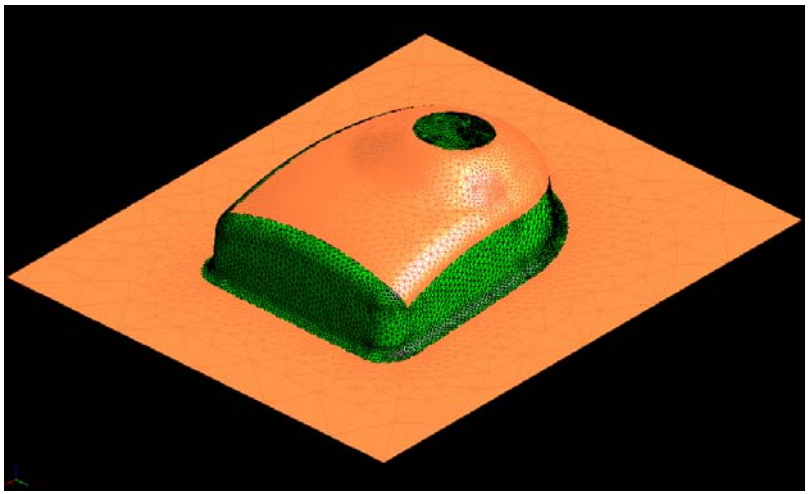


STL/Mesh/Adaptive Mesh

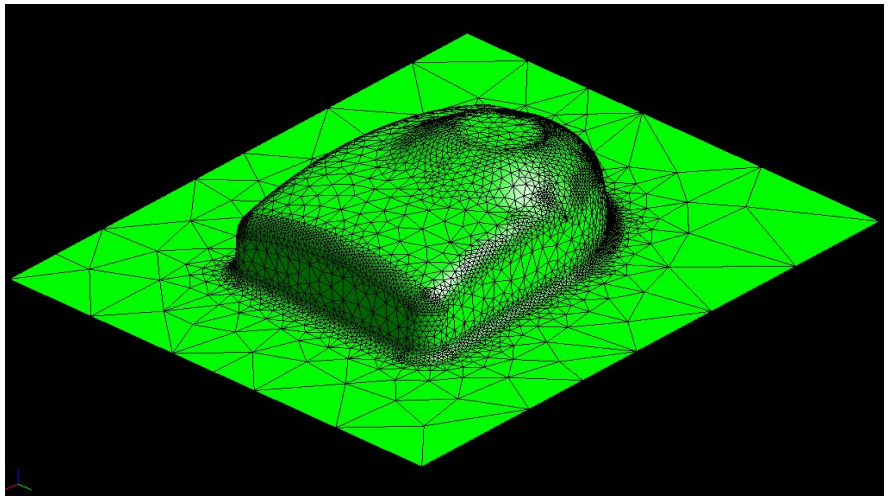
STL Mesh



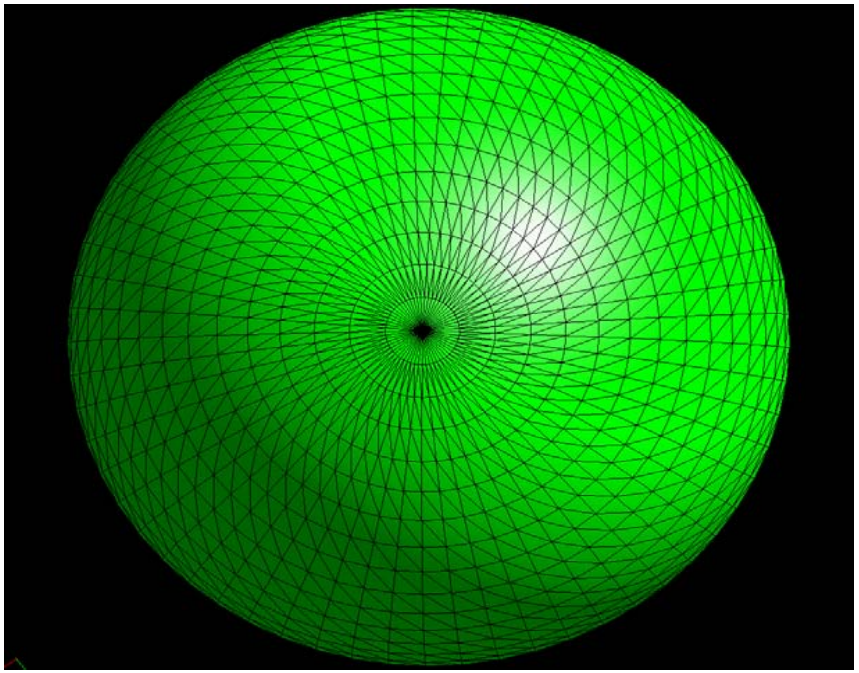
Mesh Face per Face



Adaptive Mesh

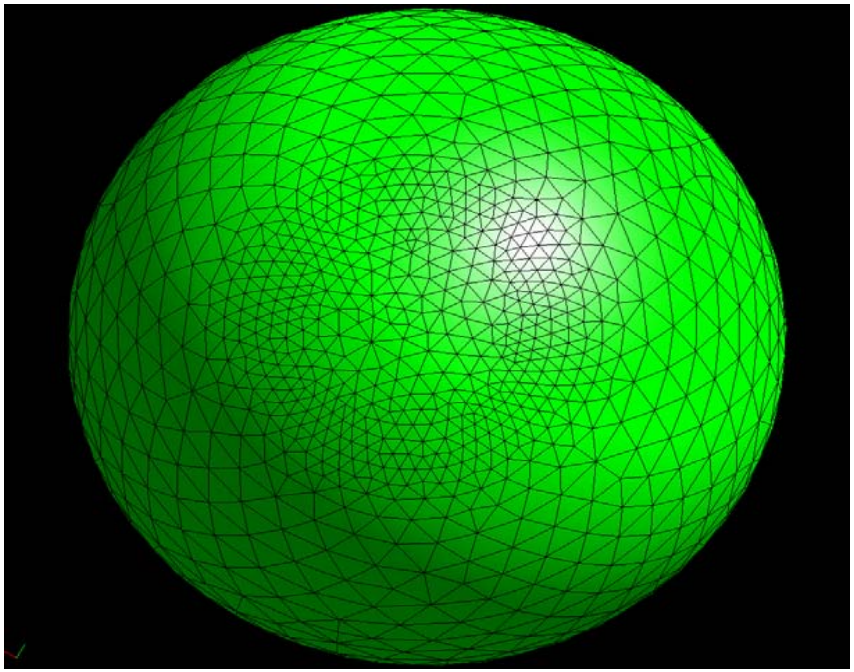


STL/Adaptive Mesh

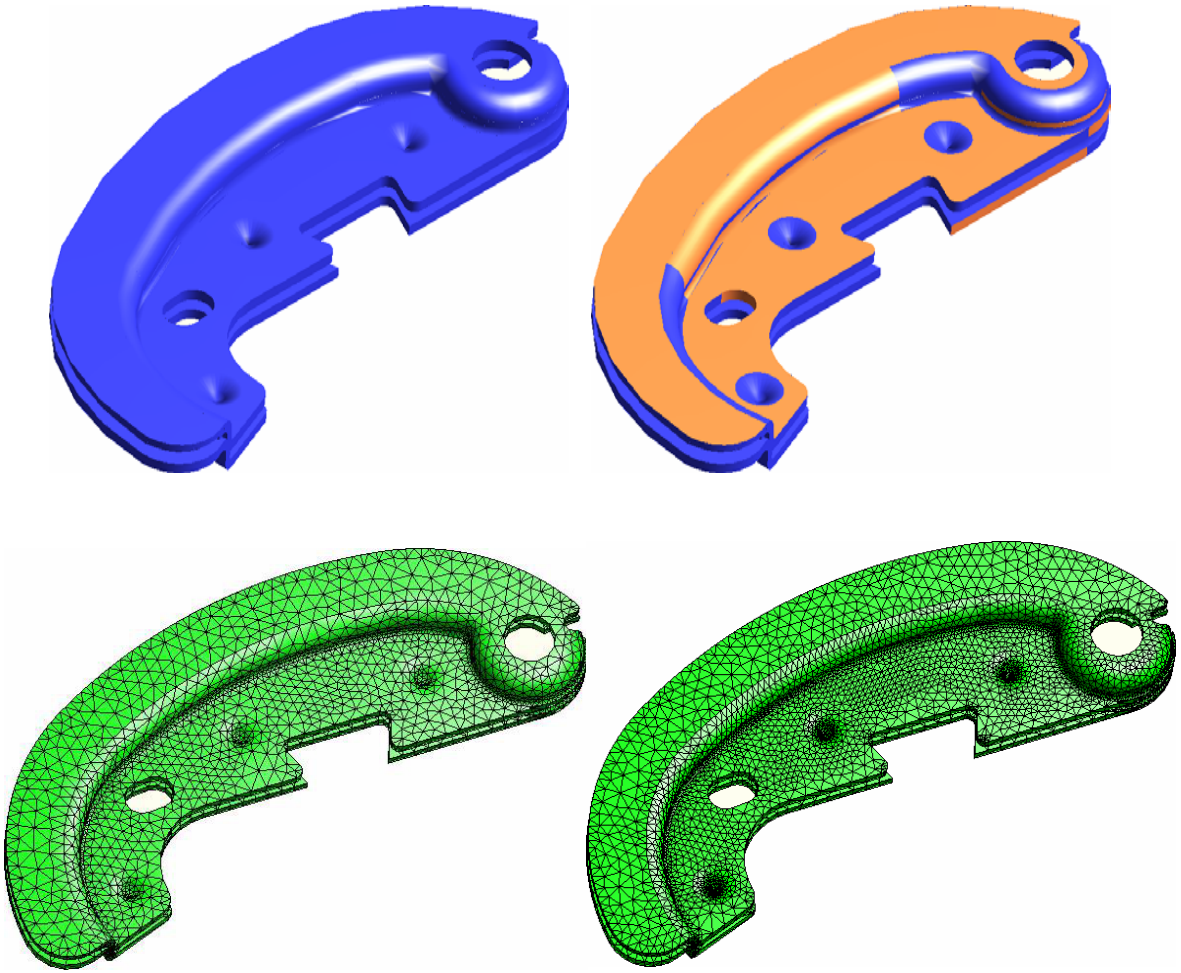


STL Mesh

Adaptive Mesh

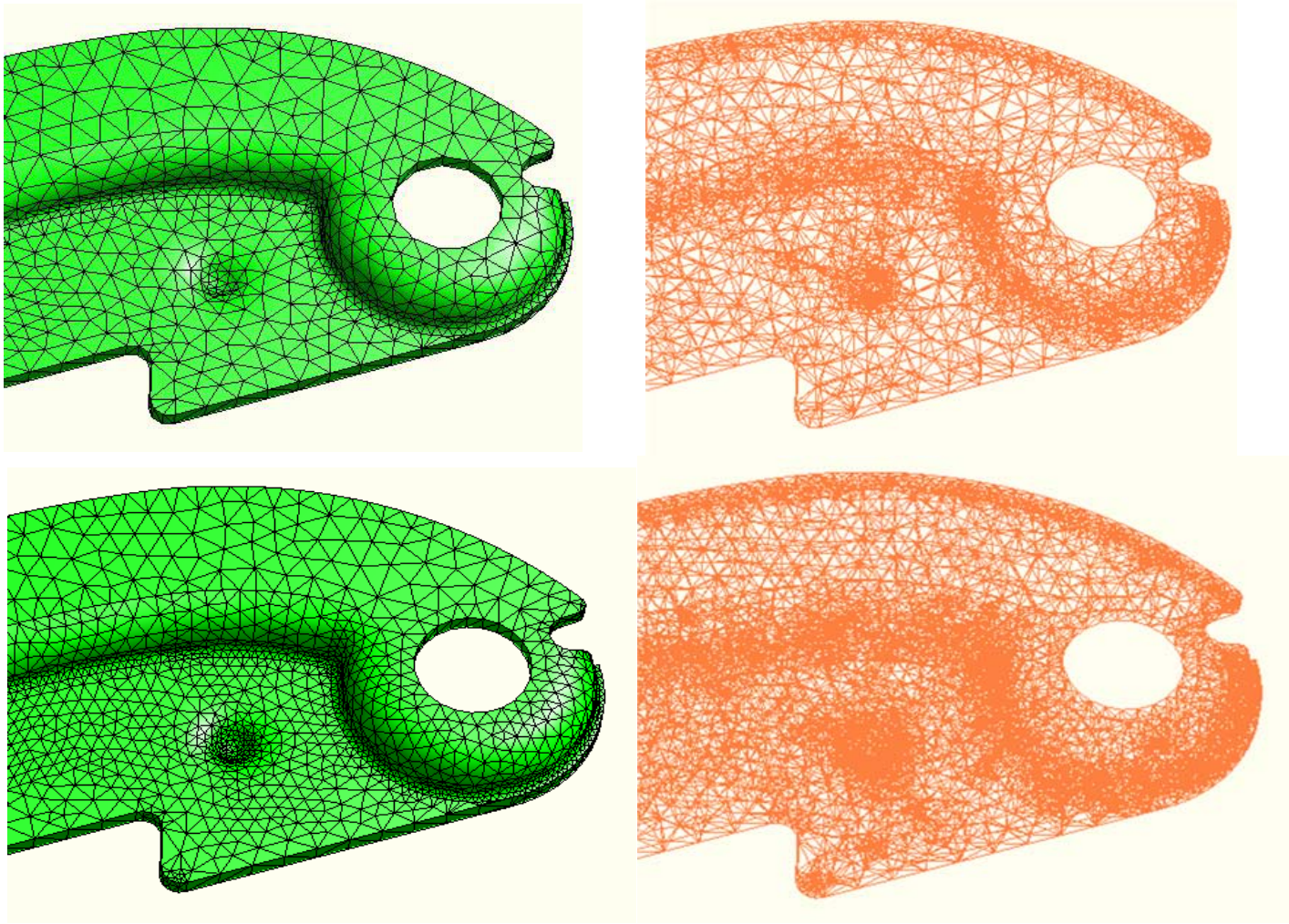


CAD/Adaptive Meshes



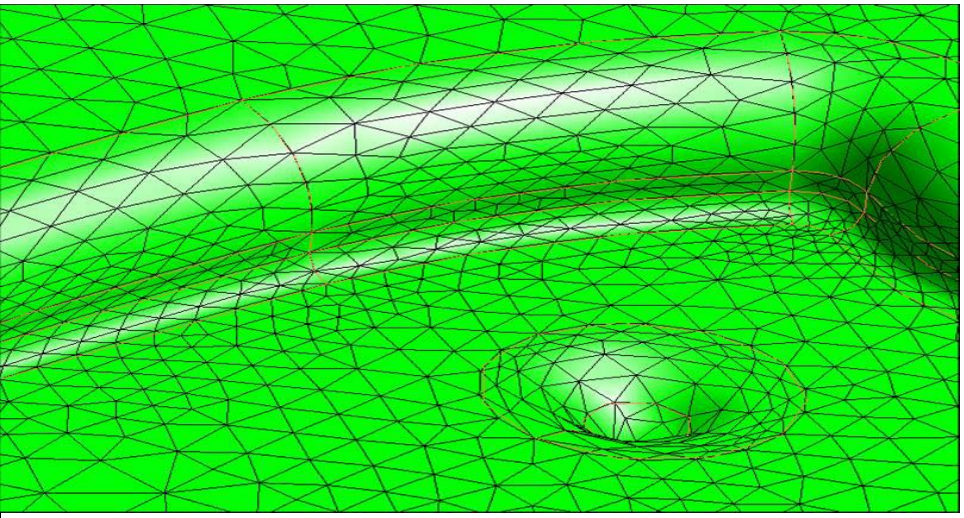
Adaptive Mesh

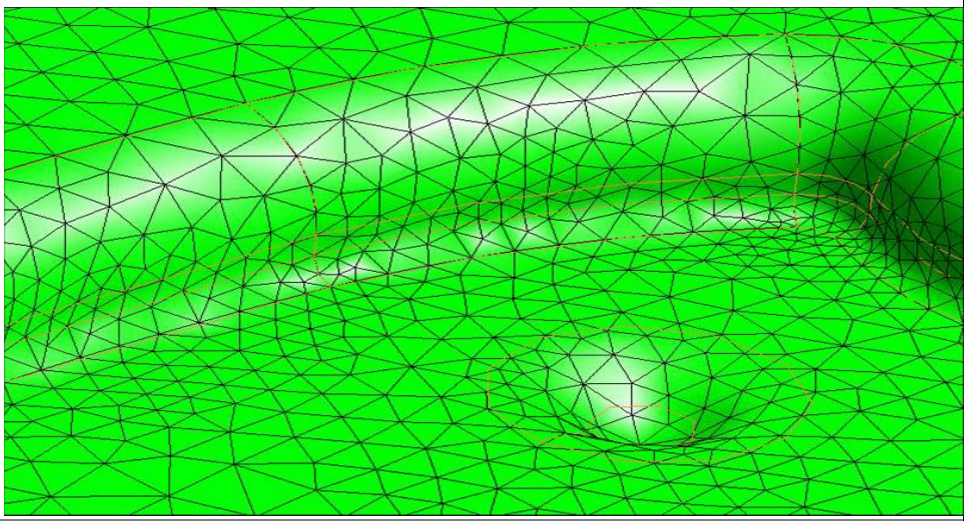
Mesh Details



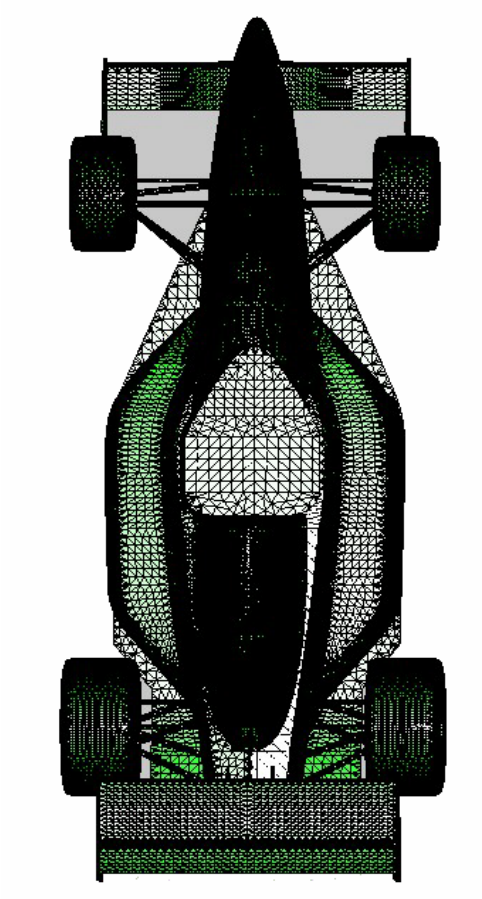
Adaptive Mesh

Feature Lines

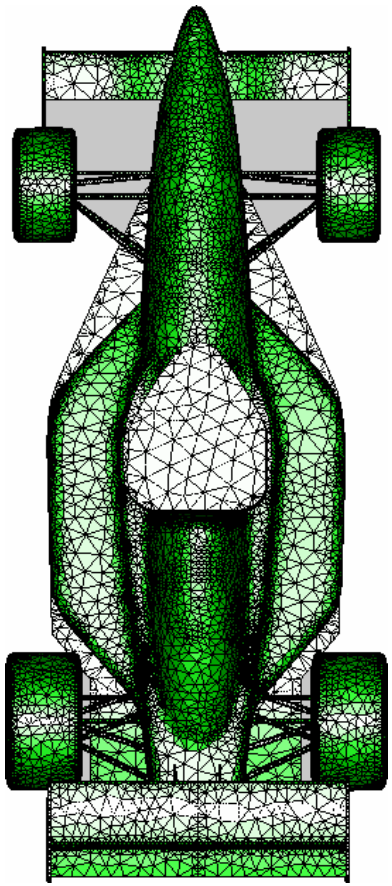




Surface Remeshing

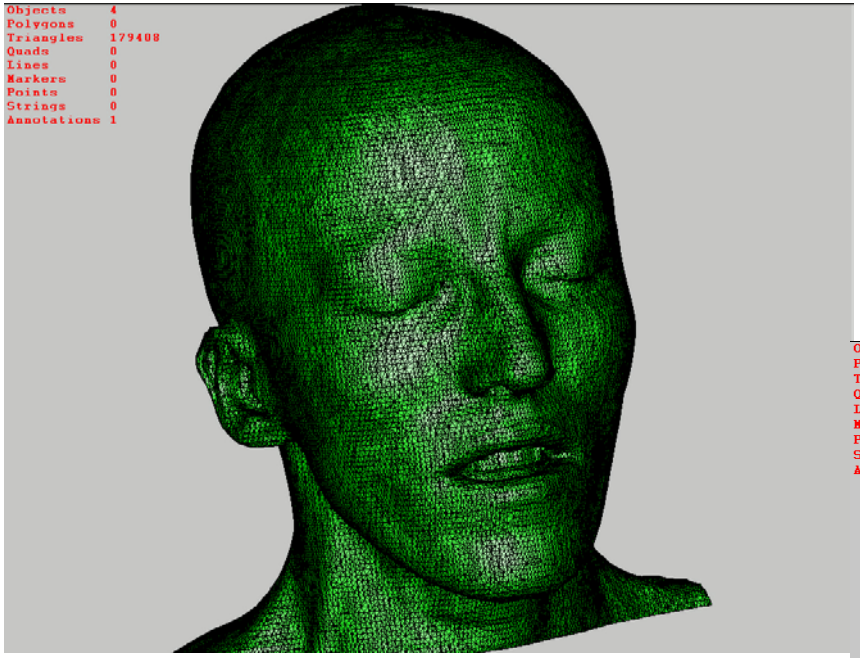


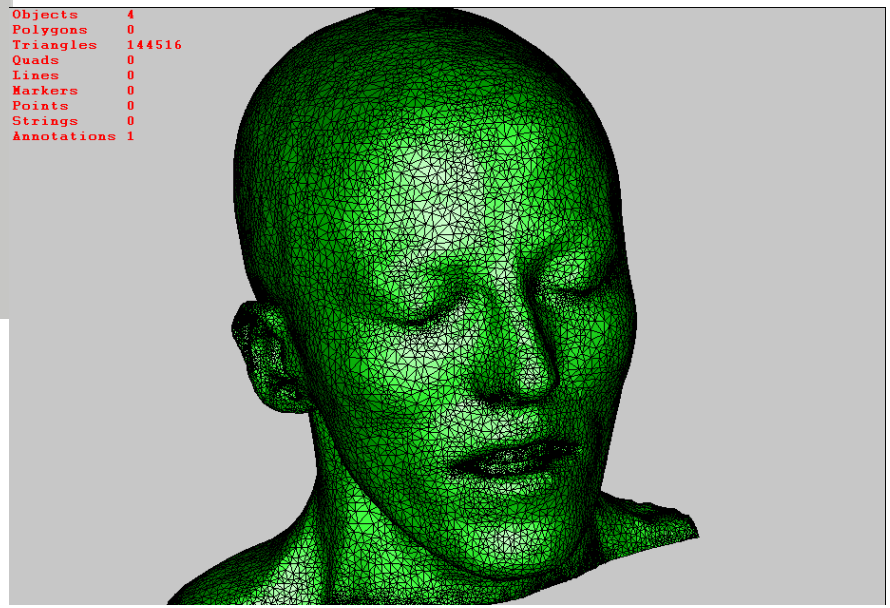
Initial model



Simplified model

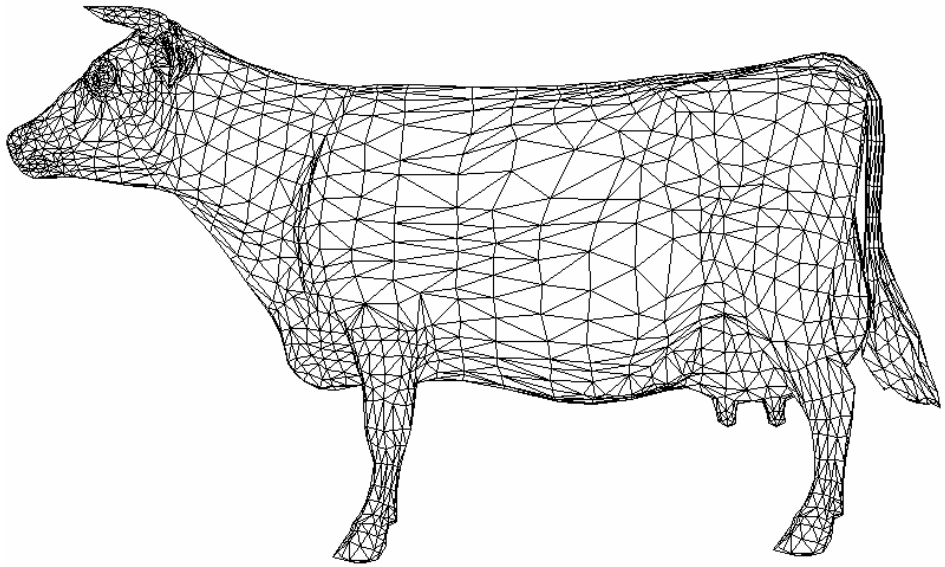
Surface Remeshing





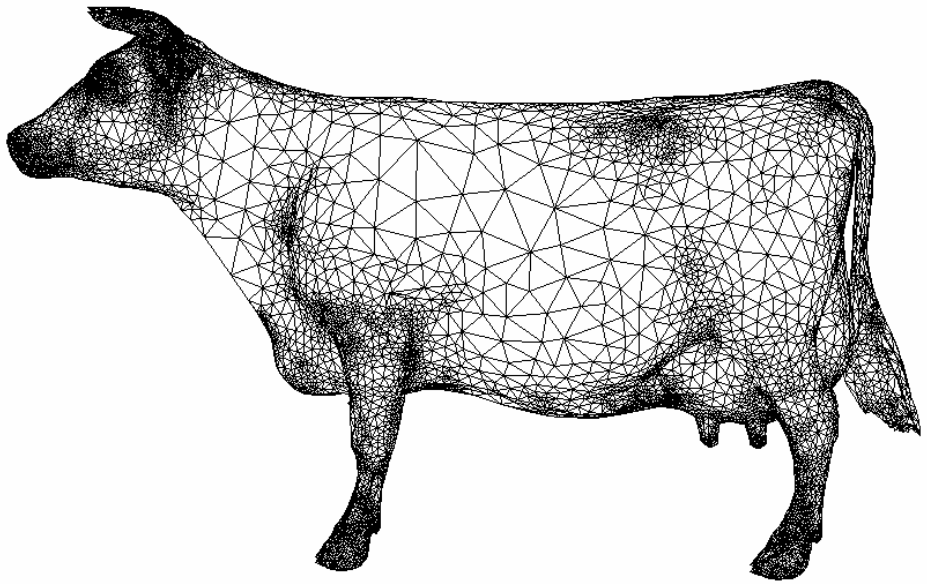
Adaptive Mesh

Surface Remeshing



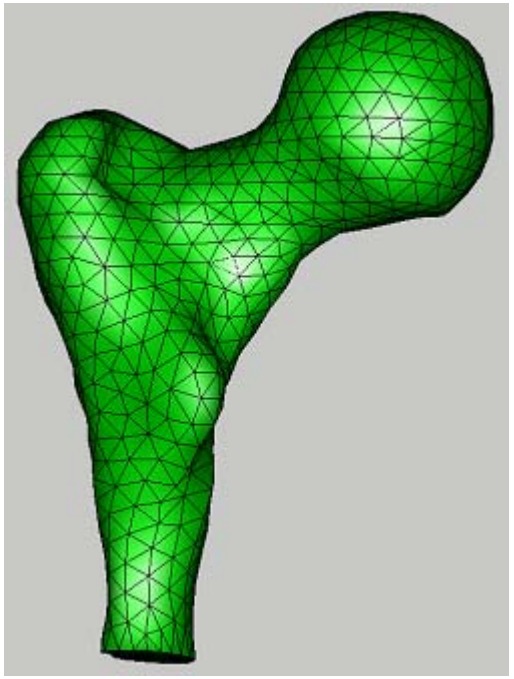
Initial Mesh

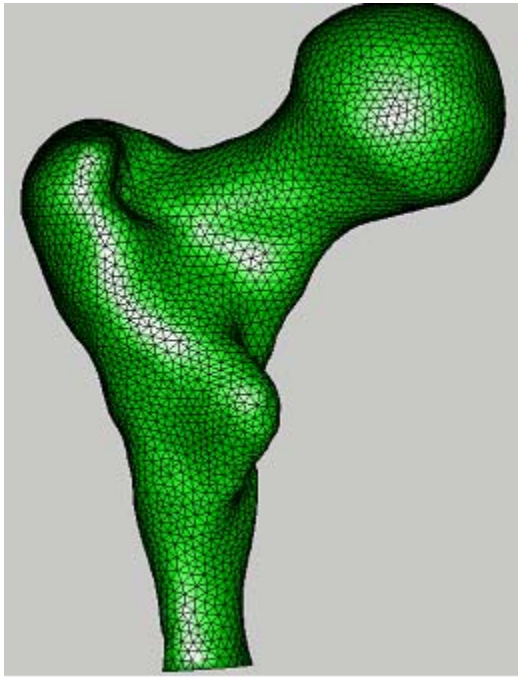
Adaptive Mesh

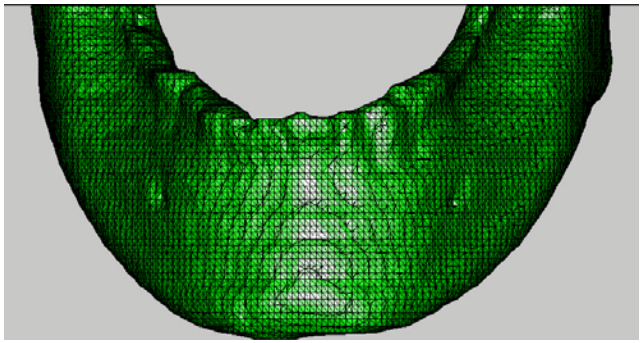


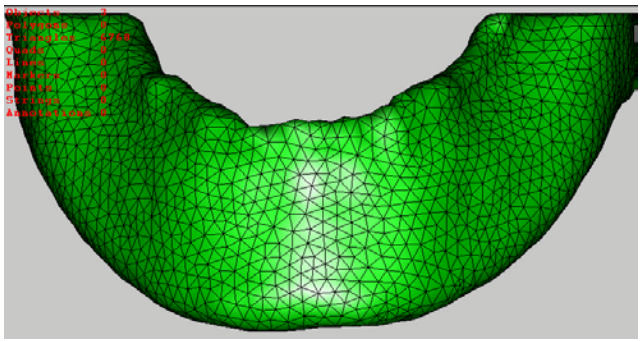
Adaptive Mesh

Surface Remeshing



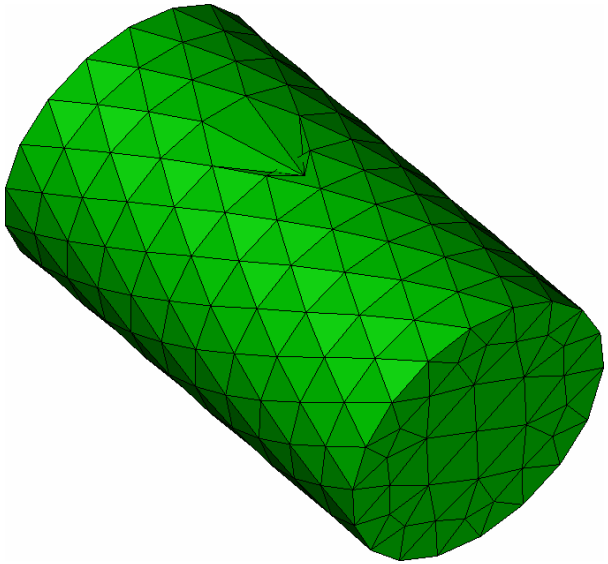


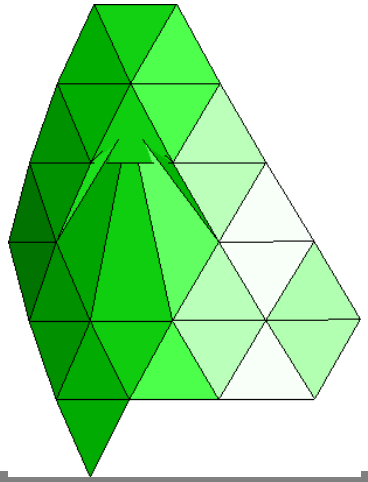


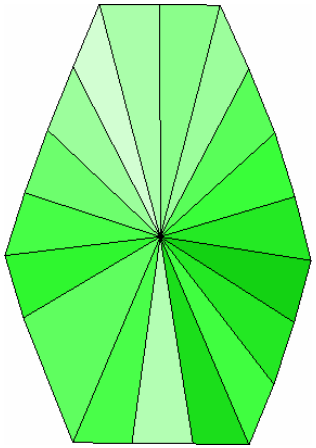


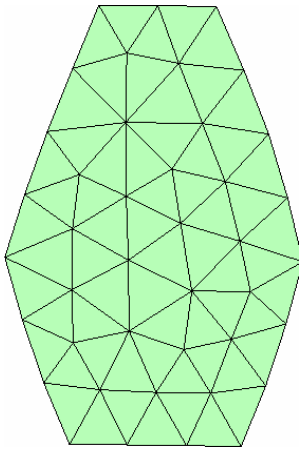
Adaptive Mesh

Repairing Surface Mesh









Demonstrations